Math

Algebra 2

High School Teacher Edition

Oak Meadow

Algebra 2

Teacher Edition

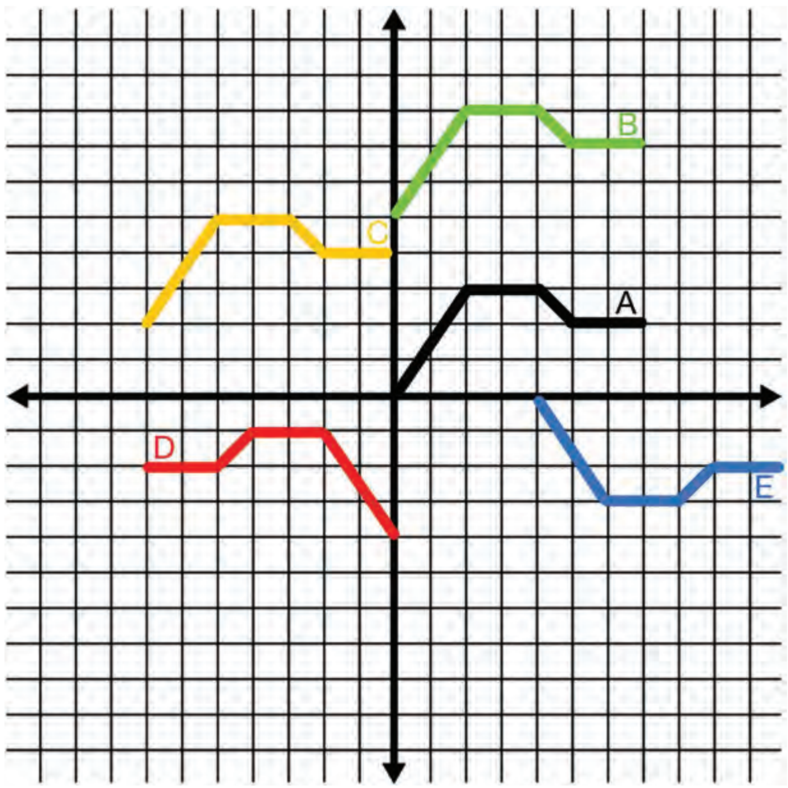
Oak Meadow

Oak Meadow, Inc.
Post Office Box 615
Putney, Vermont 05346
oakmeadow.com

Lesson 3

Part 1: Graphs and Functions

Exploratory Activity



ASSIGNMENT CHECKLIST

- ☐ Complete the exploratory activity.
- ☐ Read sections 3.1–3.5 and complete the assigned problems.
- ☐ Choose an activity to complete:

  Activity A: Integrated Review

  Activity B: Discounts and Composite Functions

  Activity C: Error Analysis of Domain and Range

  Activity D: NCTM Puzzle—Linear Relationships and Composition of Functions

What do you notice?

Students should make observations about the graphs (i.e., they are all the same shape, just oriented differently, etc.)

What do you wonder about?

Students should pose questions they have. For instance, they might wonder what kind of graph this is, how the graphs are related, how their equations are related, what it means to reflect or transform a graph, etc.

Lesson Introduction

Suggested time: 1 week

What is a function? How can its graph move or reflect? What is domain and range? These are all questions that you will explore in this lesson as you dive deep into different types of **functions and their graphs**. You will see how functions and graphs help us model and solve real-world problems, explore different transformations and how they affect the graphs of functions, and look at how a function can be restricted in its domain and range.

Learning Objectives

Use the checklist below to track how your skills are developing over time, and identify skills that need more practice as you work through part 1 and part 2 of this lesson.

Skills	Notes
Graph points and lines on the coordinate plane	
Find the slope and y-intercept of a line from an equation or set of points	
Graph nonlinear functions with transformations	
Find the equation of a line given specific conditions	
Graph 2D inequalities and their solution set on the coordinate plane	
Identify if a relation is a function, and the domain and range of a function	
Interpret a function and use it to make predictions	

Exercise Sets

Read the following sections, and then complete the accompanying problem sets. Plan to complete a portion each day. If you have online access to MyMathLab, you can watch the instructional videos as well.

As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 3.1, “Graphing Equations” (117), and then complete the following problems in Exercise Set 3.1.
   - ☐ 1–53 EO odd
2. Read section 3.2, “Introductions to Functions” (132), and then complete the following problems in Exercise Set 3.2.
   - ☐ 1, 5, 11, 13, 17, 21
   - ☐ 23–67 EO odd
   - ☐ 71–85 EO odd
   - ☐ 91, 99
3. Read section 3.3, “Graphing Linear Functions” (143), and then complete the following problems in Exercise Set 3.3.
   - ☐ 1–33 odd
   - ☐ Review and Preview 61–65 odd
4. Read section 3.4, “The Slope of a Line” (154), and then complete the following problems in Exercise Set 3.4.
   - ☐ 1, 9, 11
   - ☐ 19–43 odd
   - ☐ 45–77 EO odd
5. Read section 3.5, “Equations of Lines” (164), and then complete the following problems in Exercise Set 3.5.
   - ☐ 3, 5, 11, 15, 19, 25, 29, 31
   - ☐ 35–39 odd
   - ☐ 41, 43, 49, 51, 55, 67, 69, 71, 77

Activities

Choose one of the following activities to complete.

- Activity A: Integrated Review
- Activity B: Discounts and Composite Functions
- Activity C: Error Analysis of Domain and Range
- Activity D: NCTM Puzzle—Linear Relationships and Composition of Functions

Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

Activity A: Integrated Review

In your textbook, complete the integrated review on page 167, problems 1–23 odd. Show all your work. Check your answers in the textbook appendix, and make corrections to any problems you missed.

Activity B: Discounts and Composite Functions

Complete the following activity to explore functions and linear equations in real-world behavioral finance. This activity covers sales price and discounts, function notation, composition of functions, and writing equations.

1. On the following worksheet, read the introduction and watch the video linked in the upper right corner. You can either print this document and complete your work directly on it or use a separate sheet of paper to answer the questions.

"Math: Discounts and Composite Functions"

(All online resources can be accessed through the Oak Meadow website at oakmeadow.com/curriculum-links.)

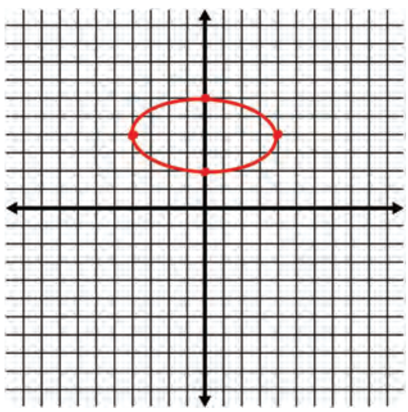
2. Study the example problem in Part I, and then complete the practice problems in Part II and the reflection questions in Part III. Show all your work.

You can find the answer key at oakmeadow.com/answer-keys.

Activity C: Error Analysis of Domain and Range

Six students found the domain and range for the following graphs. Examine their work and analyze their answers.

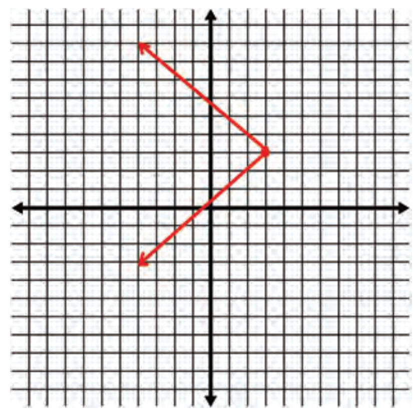
1.



Domain: All real numbers

Range: All real numbers

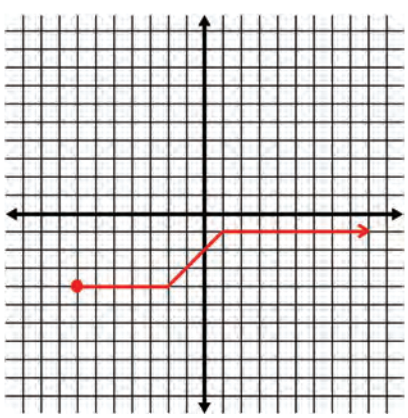
2.



Domain: $-4 \leq x \leq 3$

Range: $-3 \leq y \leq 9$

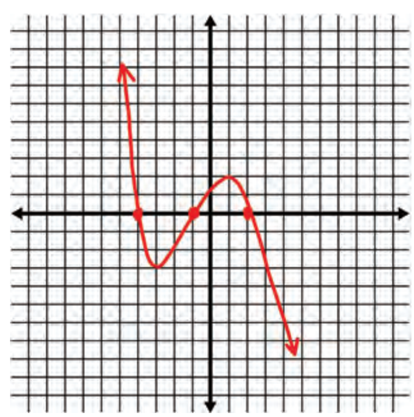
3.



Domain: All real numbers

Range: All real numbers

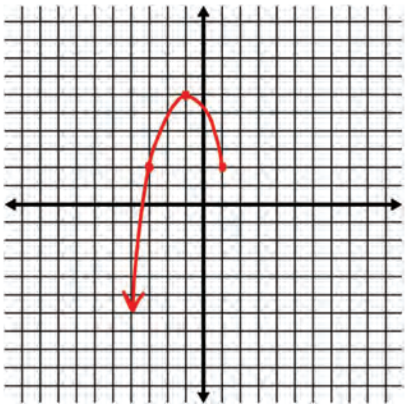
4.



Domain: All real numbers

Range: $-3 \leq y \leq 2$

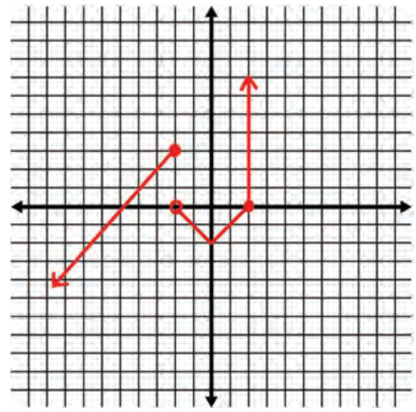
5.



Domain: $-3 \le x \le 1$

Range: $2 \le y \le 6$

6.



Domain: $x \le 2$

Range: $-2 \le y \le 0$, $y=3$

1. Correct each student's work. Provide the correct domain and range for each graph.

   1. Domain: $2 \le x \le 6$ **Range: $-4 \le y \le 4$**

   2. Domain: $x \le 3$ **Range: All real numbers**

   3. Domain: $x \ge -7$ **Range: $-4 \le y \le -1$**

   4. Domain: All real numbers **Range: All real numbers**

   5. Domain: $x \le 1$ **Range: $y \le 6$**

   6. Domain: $x \le 2$ **Range: All real numbers**

2. What misconceptions did you observe in each student's work?

   Answers will vary. Students should observe misconceptions such as seeing arrows on graphs as endpoints instead of understanding that arrows at the ends of graphs mean the values continue infinitely.

3. What advice would you give to help someone properly define domain and range to avoid making these mistakes?

   Answers will vary. Students should offer advice around how to properly define the domain and range from a graph.

Activity D: NCTM Puzzle—Linear Relationships and Composition of Functions

Complete one of the following puzzles. Show all your work supporting your answers.

1. Find the *x*- and *y*-intercepts of the perpendicular bisector of a segment whose endpoints are (1, 1) and (7, 9).

   Answer: *y*-intercept (0, 8), *x*-intercept $\left(\frac{32}{3}, 0\right)$

   Students should first calculate the slope between the given points, and then take the opposite reciprocal to find the slope of the perpendicular bisector.

   $m = \frac{9-1}{7-1} = \frac{8}{6} = \frac{4}{3}$

So, the slope of the perpendicular bisector is $m = -\frac{3}{4}$

Next, calculate the midpoint between the given points to find a point on the perpendicular bisector.

$M = \left(\frac{1+7}{2}, \frac{1+9}{2}\right) = (4, 5)$

Using the point-slope form of an equation of a line with the perpendicular slope and midpoint, and simplifying into slope-intercept form, gives the *y*-intercept (0, 8).

$y - 5 = -\frac{3}{4}(x - 4)$

$y - 5 = -\frac{3}{4}x + 3$

$y = -\frac{3}{4}x + 8$

To find the *x*-intercept, substitute 0 in for *y* in the linear equation and solve for *x*.

$0 = -\frac{3}{4}x + 8$

$-8 = -\frac{3}{4}x$

$x = \frac{32}{3}$

2. Find the slope of the linear function, *f*, given that for all real *x*,

$$f(x - 3) = f(x) + 24$$

Answer: $m = -8$

Let $x = 3$. Then we have $f(0) = f(3) + 24$ or $f(0) - f(3) = 24$. Recognizing that $f(0)$ and $f(3)$ represent the *y*-value outputs for the corresponding *x*-value inputs, we can write the slope of the function as follows:

$m = \frac{f(0) - f(3)}{0 - 3} = -\frac{24}{3} = -8$

SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 3.1–3.5 (showing handwritten computations and corrections)
- Choice of activity (labeled with the title of the activity):
  - Activity A: Integrated Review
  - Activity B: Discounts and Composite Functions
  - Activity C: Error Analysis of Domain and Range
  - Activity D: NCTM Puzzle—Linear Relationships and Composition of Functions

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson

3

Part 2: Graphs and Functions

Lesson Introduction

Suggested time: 1 week

Lesson 3 continues with part 2. Refer to part 1 for learning objectives.

Exercise Sets

Read the following sections, and then complete the accompanying problem sets. Plan to complete a portion each day. If you have online access to MyMathLab, you can watch the instructional videos as well.

As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 3.6, "Graphing Piecewise-Defined Functions and Shifting and Reflecting Graphs of Functions" (174), and then complete the following problems in Exercise Set 3.6.
   - ☐ 1–45 EO odd
   - ☐ Review and Preview 49–52 all
   - ☐ Extension 53, 55
2. Read section 3.7, "Graphing Linear Inequalities" (179), and then complete the following problems in Exercise Set 3.7.
   - ☐ 1–57 EO odd
3. Optional: If you would like more practice, you have the option of completing the following, doing as many problems as needed.
   - ☐ Chapter 3 Review and Vocabulary Check (184)
   - ☐ Chapter 3 Standardized Test Practice (189)

ASSIGNMENT CHECKLIST

- ☐ Read sections 3.6–3.7 and complete the assigned problems.
- ☐ Complete the chapter 3 test.
- ☐ Complete the assessment test (if provided).
- ☐ Choose an activity to complete:

  Activity A: Investigate Graphing Stories and Piecewise Functions

  Activity B: Real-World Application of Systems of Linear Inequalities

  Activity C: Explore Function Transformations

  Activity D: Get Creative with Piecewise Functions

Chapter Test

1. In your textbook, complete the chapter 3 test on page 187. After completing the test, you or a supervising adult will grade it and mark the score at the top (for instance, 18/20). Then, review any mistakes and make necessary corrections.
2. For enrolled students: Complete the chapter 3 assessment test, if one has been provided.

Activities

Choose one of the following activities to complete.

- Activity A: Investigate Graphing Stories and Piecewise Functions
- Activity B: Real-World Application of Systems of Linear Inequalities
- Activity C: Explore Function Transformations
- Activity D: Get Creative with Piecewise Functions

Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

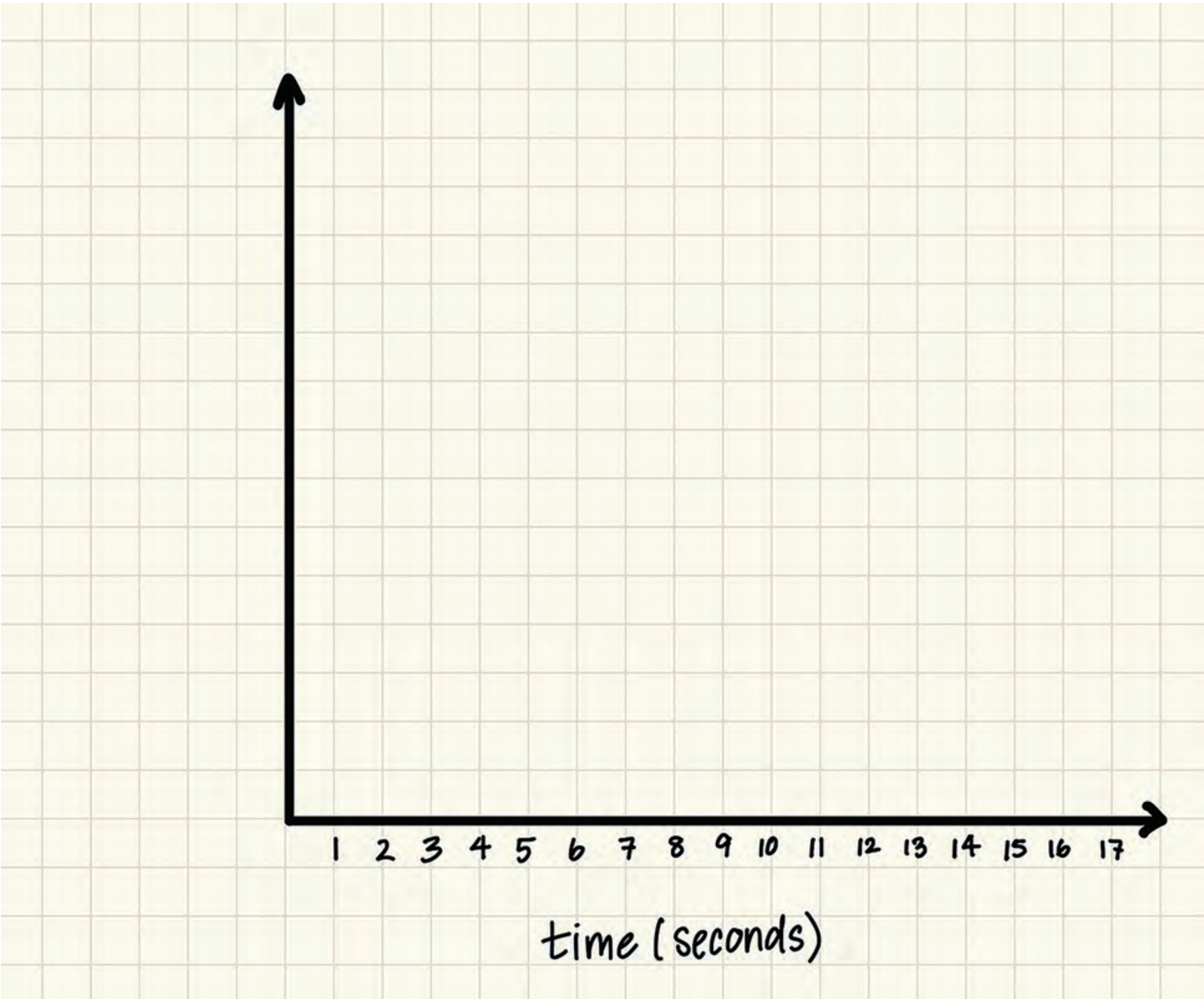
Activity A: Investigate Graphing Stories and Piecewise Functions

1. Watch the following video.

   "Graphing Stories—Balloon Length"

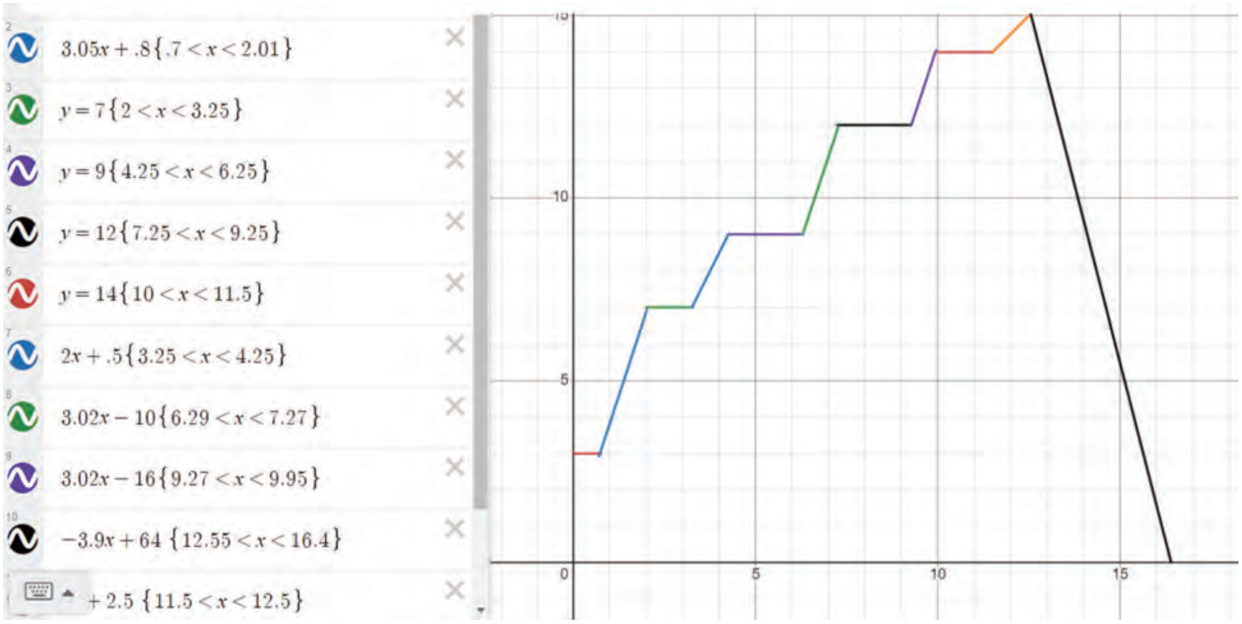
   (All online resources can be accessed through the Oak Meadow website at oakmeadow.com/curriculum-links.)

2. Label the y-axis on the graph below and follow along with the video to graph the balloon length as it is being blown up. Pause the video as needed to make your graph as accurate as possible.



3. At the end of the video, it reveals the correct graph. In a different color, draw the correct graph over your graph and label each section with what it represents.

   Students should have something very close to the following graph overlaid in a different color to their own graph. It does not need to be exact.



4. How does your graph compare?

   Students should compare and contrast their graph and the correct answer. They should examine any inconsistencies and explain their thinking.

5. What would you do differently if you were presented with graphing a situation like this again?

   They should discuss anything they would do differently next time, such as pay closer attention to the time, length, etc.

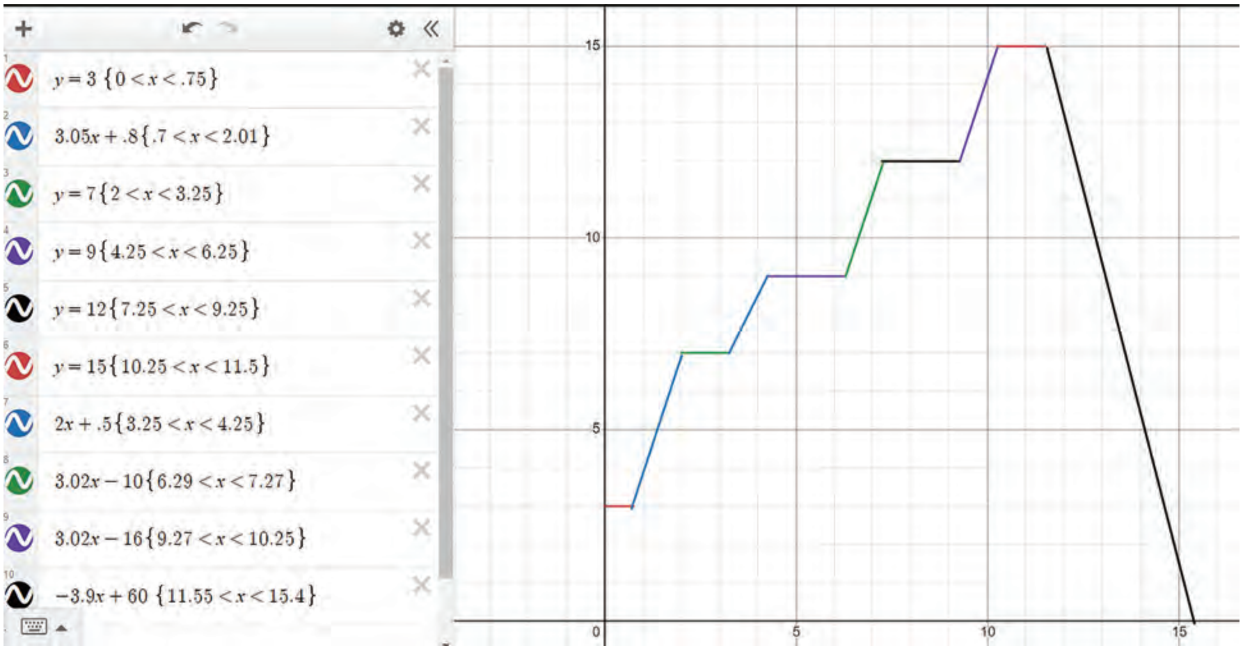
6. Finally, use the Desmos online graphing calculator (www.desmos.com/calculator) to recreate the graph. Make sure your scales on both axes are correct.

   If you need a reminder on how to restrict the domain and/or range of a function in Desmos, check out the following resource.

   "Getting Started: Inequalities and Restrictions"

   Submit your Desmos file, including all your equations along with your work for parts A through C.

The solution should resemble something close to this (it does not need to be exact). Verify the student's equations and that they properly formatted piecewise functions with appropriate domains.



Activity B: Real-World Application of Systems of Linear Inequalities

A high school soccer team is raising money to buy new uniforms and equipment. They are holding a fundraiser where they are selling chocolate boxes and flower bouquets. The chocolates are sold for \$5 a box and the flowers are sold for \$6 a bouquet. Altogether, the soccer team has 200 items to sell and is hoping to make at least \$500.

1. Write a system of inequalities to model this situation. Make sure to define your variables.

   x = boxes of chocolate

   y = flower bouquets

   $5x + 6y \geq 500$

   $x + y \leq 200$

   $x \geq 0$

   $y \geq 0$

2. Graph both equations by hand or use the Desmos online graphing calculator (www.desmos.com/calculator). Make sure to restrict the domain and range to represent this situation.

   (All online resources can be accessed through the Oak Meadow website at oakmeadow.com/curriculum-links.)

Students should create a graph restricted to the positive values in Quadrant 1.

3. Name two different outcomes where the team sells out of items.

   Answers will vary. Students should name points on the green line in the graph above, which represents all 200 items sold. For example: (100, 100), (200, 0), (140, 160), etc.

4. Name two different outcomes where the team does not meet their fundraising goal.

   Answers will vary. Students should name points in the green region in the graph above, which represents a profit less than $500. For example: (20, 20), (40, 20), (20, 40), etc.

5. Name two different outcomes where the team exceeds their fundraising goal but does not sell out of items.

   Answers will vary. Students should name points in the overlap region in the graph above, which represents a profit more than $500 but with less than 200 items sold. For example: (100, 60), (20, 120), (80, 80), etc.

6. What does the point (200, 100) represent?

   Students should describe what the purple region in the graph above represents. This region contains the point (200, 100), and it represents the area in which the profit has exceeded the fundraising goal but the team has sold out of items. Because of the finite supply of chocolate and flowers, it is not actually possible to be in the purple region in this scenario.

Activity C: Explore Function Transformations

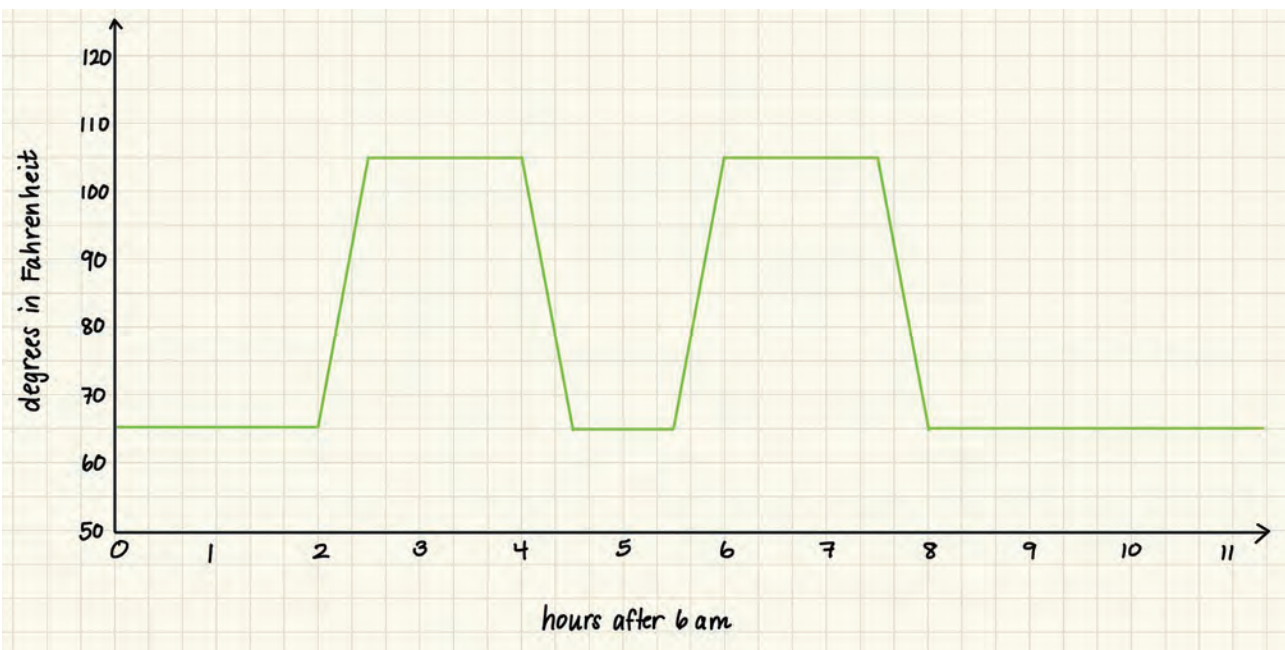
Explore the following GeoGebra applets to solidify your understanding of transformations of functions. Complete at least three examples on each applet before moving on to the activity.

> "Graphing Absolute Value Functions with Transformations"
>
> "Writing Absolute Value Functions from Graphs"

(All online resources can be accessed through the Oak Meadow website at oakmeadow.com/curriculum-links.)

At a hot yoga studio, the temperature of the room gets raised to 105 degrees Fahrenheit for each yoga class. To save on the energy bill, the studio owner only turns the heat up half an hour before each class and turns the heat down to 65 degrees in between classes. Each class is 90 minutes long. (For our purposes, we'll assume the temperature drops and rises in a matter of minutes, much more quickly than it would in reality.)

On Saturdays, the yoga studio offers a class at 8:30 a.m. and noon. The following graph models the temperature in the yoga studio on Saturdays.



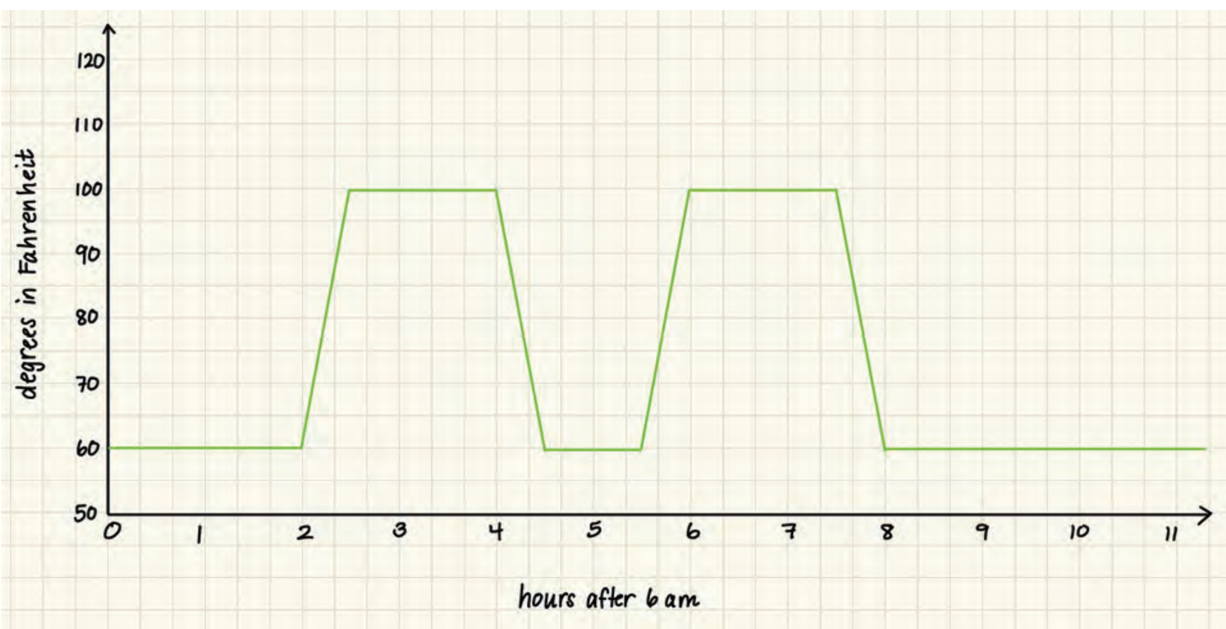
1. On Mondays, the studio offers classes at 10:00 a.m. and 1:30 p.m. What would the graph of the temperature in the yoga studio on Mondays look like?

   Students should graph the function shifted an hour and a half to the right.

2. The temperature in the yoga studio on Saturdays can be represented with the function *H*(*t*). What would the function for Mondays be? Explain your answer.

   The function for Mondays would be *H*(*t* – 1.5). This is because the schedule is shifted an hour and a half later in the day, which translates to a horizontal shift to the right. This is represented by –1.5 inside the function.

3. The following graph represents the temperature in the studio on a specific Saturday in January. What information can be gathered from this graph?



Students should gather from this graph that the starting temperature in the studio is 5 degrees cooler than usual. This could be due to a particularly cold day outside or a malfunctioning thermostat. The result is that the yoga studio only reaches 100 degrees on this particular day.

4. How would the function $H(t)$ be altered to represent this situation? Explain your answer.

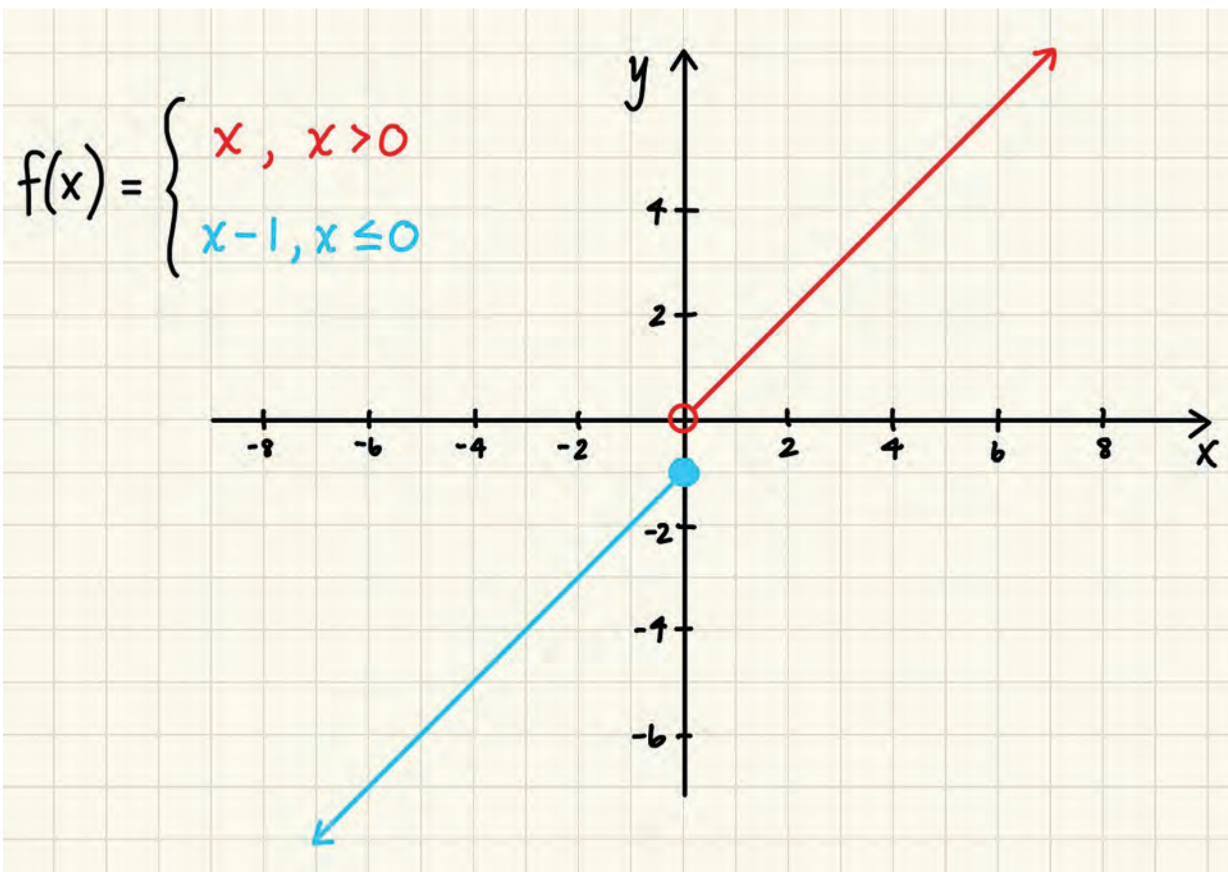
The function for this day would be $H(t) - 5$. This is because the temperature is 5 degrees cooler, which translates to a vertical shift down. This is represented by –5 outside the function.

Activity D: Get Creative with Piecewise Functions

Create a piecewise function that satisfies each of the following conditions. Include the piecewise equation and the graph for each. Create graphs by hand or use the Desmos online graphing calculator (www.desmos.com/calculator). Ensure all your functions meet the criteria to be a function.

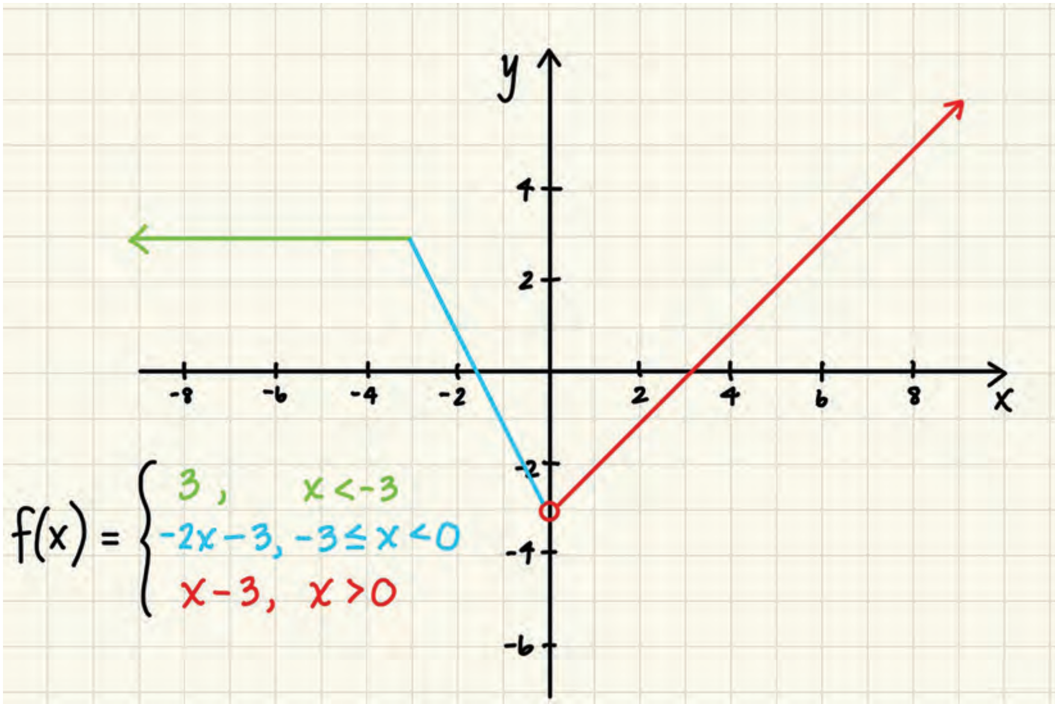
1. Two parts; Domain: All real numbers; Range: All real numbers.

Answers will vary. Verify student's work and that all graphs/equations represent functions. Example solutions are below.



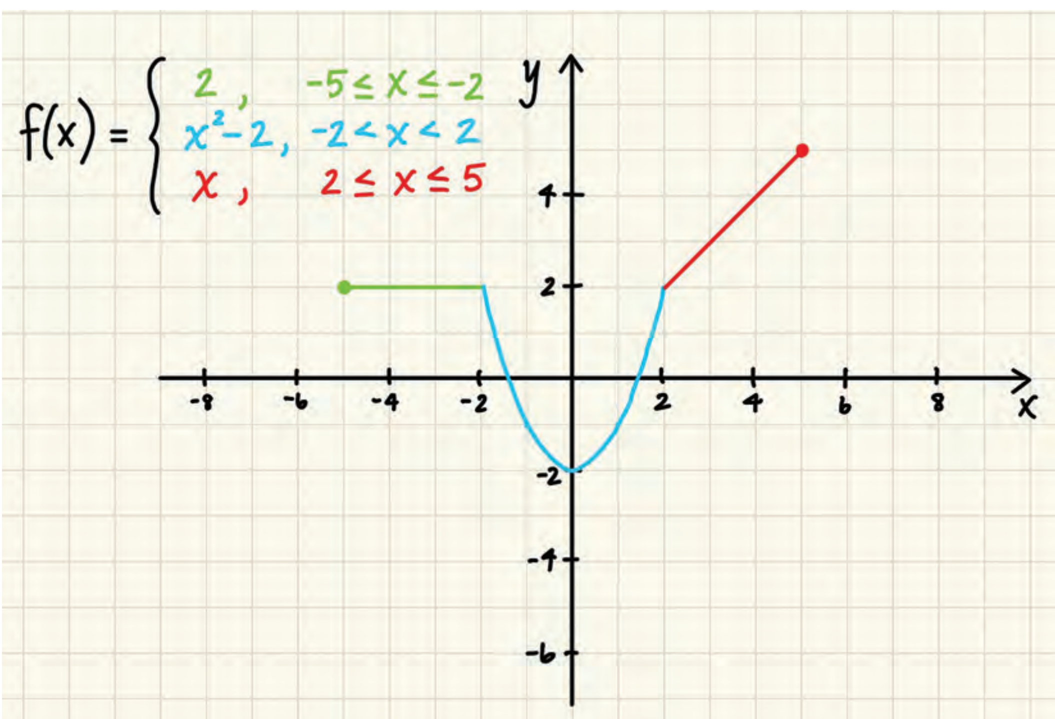
2. Three parts; Domain: All real numbers except 0; Range: $y > -3$.

   Example:



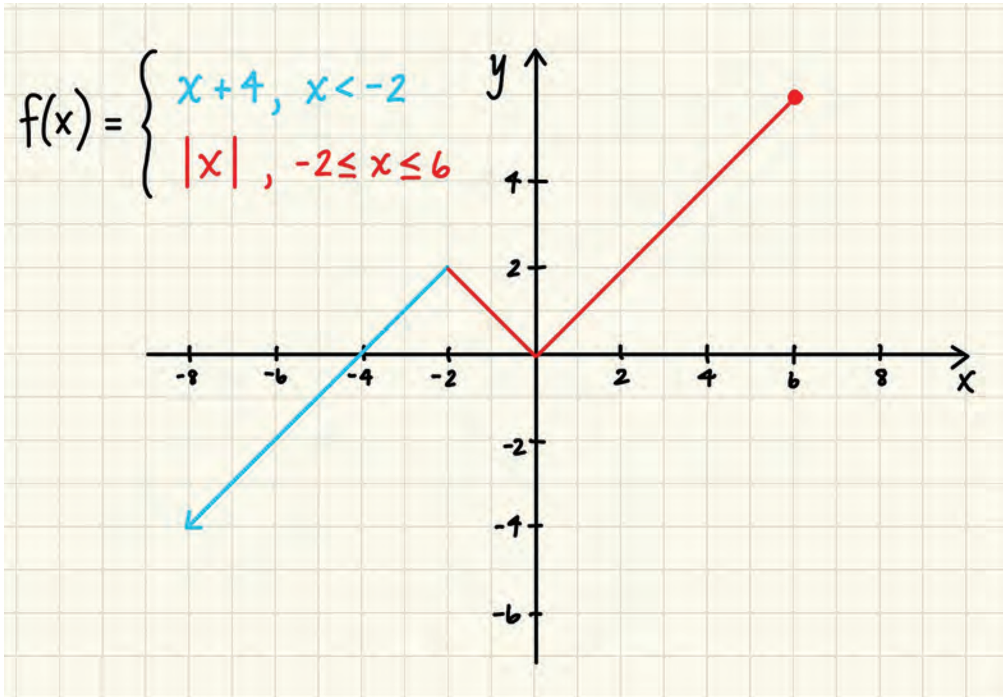
3. At least one quadratic part; Domain: $-5 \le x \le 5$; Range: $-2 \le y \le 5$.

   Example:



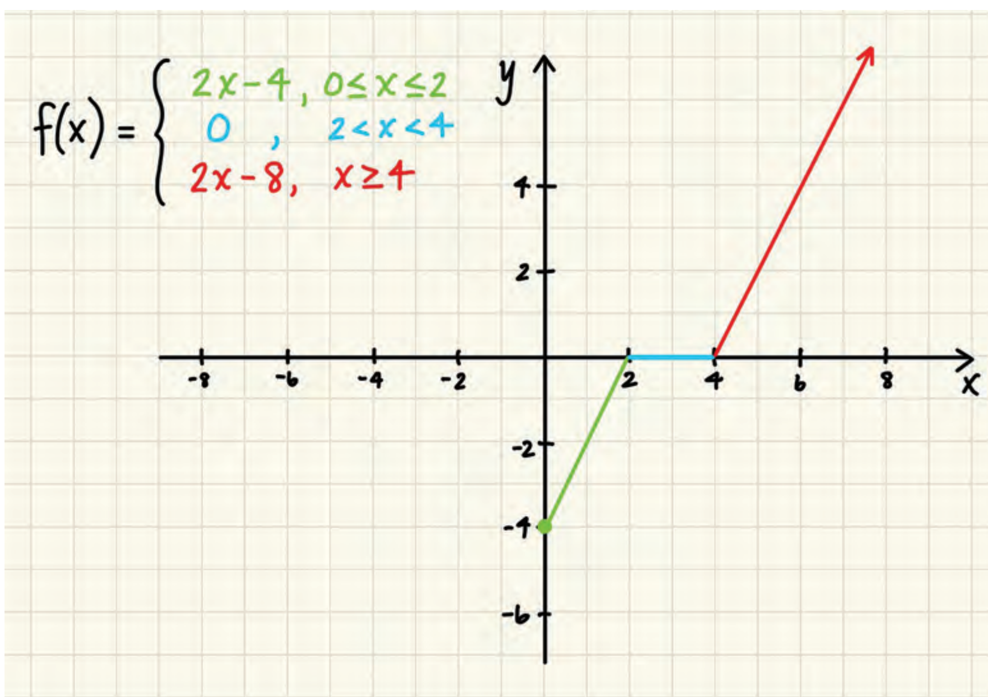
4. At least one absolute value part; Domain: $x \leq 6$; Range: $y \leq 6$.

Example:



5. Your choice; Domain: $x \geq 0$; Range: $y \geq -4$.

Example:



SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 3.6–3.7 (showing handwritten computations and corrections)
- Chapter 3 test
- Chapter 3 assessment test (if one has been provided)
- Choice of activity (labeled with the title of the activity):
    - Activity A: Investigate Graphing Stories and Piecewise Functions
    - Activity B: Real-World Application of Systems of Linear Inequalities
    - Activity C: Explore Function Transformations
    - Activity D: Get Creative with Piecewise Functions

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson

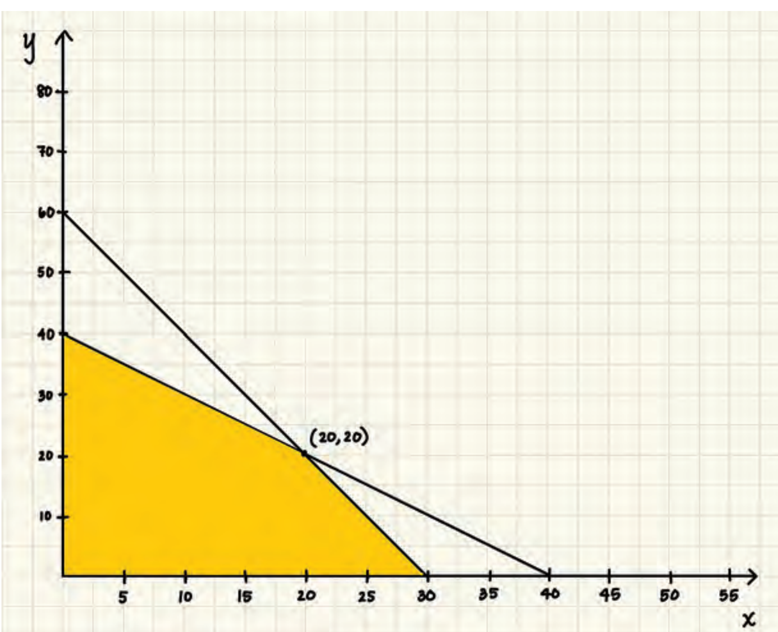
4

Part 1: Systems of Equations

Exploratory Activity

A farmer sells cantaloupes (x) and grapefruits (y) at the farmers market. She carries the produce in two different containers. One container can carry up to 40 pounds of produce. The other container is larger and can fit twice as many cantaloupes as the first container and can carry up to 60 pounds of produce total. Assuming the farmer takes only cantaloupes and grapefruits to the farmers market, this situation can be represented by the following system of inequalities and its graph:

$$x + y \leq 40$$
$$2x + y \leq 60$$
$$x \geq 0$$
$$y \geq 0$$



ASSIGNMENT CHECKLIST

- ☐ Complete the exploratory activity.
- ☐ Read sections 4.1–4.3 and complete the assigned problems.
- ☐ Choose an activity to complete:

  Activity A: Integrated Review

  Activity B: Citizen Math—Systems of Linear Equations in Society

  Activity C: Which One Doesn't Fit? Solutions to Linear Systems

  Activity D: NCTM Puzzle—Systems of Linear Equations

Analyze the system and its graph. What does the point (20, 20) represent?

This point represents a solution where both inequalities are at their maximum. Both containers would be at their maximum weight with 20 cantaloupes and 20 grapefruits.

What does a point inside the yellow region represent?

Any point in this region will satisfy both inequalities. If the point falls on the border, one of the inequalities would be maximized but not the other. If the point is fully inside the yellow region, both inequalities will be met without being maximized.

How does a point outside the yellow region, such as (5, 40), differ from a point such as (30, 20)?

A point that is fully outside both linear regions like (30, 20) will not satisfy either inequality. At this point, both containers would have surpassed their maximum weight. A point like (5, 40) is outside the yellow region, but it still satisfies one of the inequalities (the second one), so only one container would have surpassed its weight limit.

Lesson Introduction

Suggested time: 1.5 weeks

Systems of equations and inequalities allow us to model many types of real-life scenarios to help with decision-making or analyze potential outcomes. Systems of equations and inequalities can outline the constraints on decisions and help us see where the best solution lies. They can help us maximize profit, minimize cost, and determine the best way to allocate resources. In this lesson, you will analyze systems and their graphs and learn how to interpret their constraints in real-life contexts.

Learning Objectives

Use the checklist below to track how your skills are developing over time, and identify skills that need more practice as you work through part 1 and part 2 of this lesson.

Skills	Notes
Solve a system of equations by graphing	
Solve a system of equations by substitution or elimination	
Solve a system of equations using matrices	
Interpret and solve application problems involving systems	
Graph the solution set to a system of inequalities and find the maximum value	

Exercise Sets

Read the following sections, and then complete the accompanying problem sets. Plan to complete a portion each day. If you have online access to MyMathLab, you can watch the instructional videos as well.

As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 4.1, “Solving Systems of Linear Equations in Two Variables” (204), and then complete the following problems in Exercise Set 4.1.
   - ☐ 1–29 EO odd
   - ☐ 51–61 odd
2. Read section 4.2, “Solving Systems of Linear Equations in Three Variables” (212), and then complete the following problems in Exercise Set 4.2.
   - ☐ 1–29 EO odd
   - ☐ Extension 39, 41
3. Read section 4.3, “Systems of Linear Equation and Problem Solving” (222), and then complete the following problems in Exercise Set 4.3.
   - ☐ 1
   - ☐ 9–45 EO odd

Activities

Choose one of the following activities to complete.

- Activity A: Integrated Review
- Activity B: Citizen Math—Systems of Linear Equations in Society
- Activity C: Which One Doesn’t Fit? Solutions to Linear Systems
- Activity D: NCTM Puzzle—Systems of Linear Equations

Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

Activity A: Integrated Review

In your textbook, complete the integrated review on page 226, problems 1–21 odd. Show all your work. Check your answers in the textbook appendix, and make corrections to any problems you missed.

Activity B: Citizen Math—Systems of Linear Equations in Society

Should the government increase the minimum wage? Millions of people earn hourly wages at fast-food restaurants, but it can be difficult to determine how much to pay. On one hand, the more a restaurant pays, the more people will want to work there. On the other hand, the higher the wages, the fewer employees the restaurant will want to hire. In this activity, you'll use systems of linear equations to explore the relationship between wage and labor, analyze the economics of fast-food restaurants, and debate whether the federal government should increase the minimum wage.

1. First, watch the short video launch below to get started. (All online resources can be accessed at oakmeadow.com/curriculum-links.)

    "Wendy's Training Video Hot Drinks"

2. Think about the following questions:
   - Can you remember all the requirements for how to properly pour a hot drink at Wendy's?
   - Have you ever had a job? If so, did you receive any specific training before starting?
   - How much did your job pay? When deciding whether to take the job, how important was the amount of pay? Or, if you haven't had a job, how much would the salary factor into your decision of whether or not to take a job?
3. Complete questions 1–4 on the following worksheet. You can print it out and complete the work directly on the page or write on a separate sheet of paper.

   "Wage War"
4. Watch the following video before completing question 5.

   "One NYC Family's Struggle to Survive on a Fast Food Salary"

   You can find the answer key at oakmeadow.com/answer-keys.

Activity C: Which One Doesn't Fit? Solutions to Linear Systems

$$3x - 5y = 15$$

A	B	C
$-3x + 5y = 30$	$y = \frac{3}{5}x - 3$	$3x + 15y = 3$

Which one doesn't fit? Given the equation at the top, identify at least one reason why each of the three options doesn't fit.

Answers will vary. Students should give at least one reason why each option does not fit. Possible answers are given below.

A is the only option where there is no solution when paired with the given equation (solving gives 0 = 45, which is a false statement). It is the only option that represents a parallel line to the given equation. It is the only option that is set up to be easily solved with the elimination method when combined with the given equation.

B is the only option where there is an infinite number of solutions when paired with the given equation (solving gives 15 = 15, which is a true statement). It is the only option that represents the same line as the given equation. It is the only option that is set up to be easily solved with the substitution method when combined with the given equation.

C is the only option where there is exactly one solution when paired with the given equation (solving gives the point $\left(4, -\frac{3}{5}\right)$ as the intersection point of the two lines). It is the only option that represents a line that intersects at one point with the given equation. It is the only option

that requires one equation to be multiplied by a constant in order to be solved with the elimination method, and the only equation that requires manipulation before it can be used with the substitution method when combined with the given equation.

Activity D: NCTM Puzzle—Systems of Linear Equations

Complete one of the following puzzles. Show all your work supporting your answers.

1. For what value of k in the following linear system will the system have no solution?

   $kx - 3y = 4$

   $4x - 5y = 7$

   Answer: $\frac{12}{5}$

   Rearranging the second equation gives $y = \frac{4}{5}x - \frac{7}{5}$ **with a slope of** $\frac{4}{5}$**. For the system to have no solution, the slope of the first equation must be equal to the second equation, giving parallel lines. Rearranging the first equation gives** $y = \frac{k}{3}x - \frac{4}{3}$ **with a slope of** $\frac{k}{3}$**. Setting the two slopes equal and solving for** k **gives** $\frac{4}{5} = \frac{k}{3}$ **or** $k = \frac{12}{5}$**.**

2. Solve the system in terms of k (the x-value and y-value of the solution will both have k in them).

   Answer: $\left(\frac{-1}{5k - 12}, \frac{-16 + 7k}{-5k + 12}\right)$

   Solve the system by elimination. First, eliminate the y **terms by multiplying the top equation by 5 and the bottom equation by –3:**

   $$5kx - 15y = 20$$

   $$-12x + 15y = -21$$

   Add together:

   $$(5k - 12)x = -1$$

   $$x = \frac{-1}{5k - 12}$$

   Next, eliminate x **by multiplying the top equation by –4 and the bottom equation by** k**:**

   $$-4kx + 12y = -16$$

   $$4kx - 5ky = 7k$$

   Add together for the solution:

   $$(-5k + 12)y = -16 + 7k$$

   $$y = \frac{-16 + 7k}{-5k + 12}$$

3. Solve the following system of equations.

   $\frac{3}{x + 1} + \frac{5}{y - 2} = 1$

   $\frac{6}{x + 1} + \frac{1}{y - 2} = 5$

   Answer: $\left(\frac{1}{8}, -1\right)$

Solving with elimination, multiply the top equation by –2:

$$-2\left(\frac{3}{x+1} + \frac{5}{y-2} = 1\right) - 2$$

Add the two equations together:

$$\begin{array}{r} \frac{-6}{x+1} + \frac{-10}{y-2} = -2 \\ + \frac{6}{x+1} + \frac{1}{y-2} = 5 \\ \hline \frac{-9}{y-2} = 3 \end{array}$$

Solve for y: $-9 = 3(y - 2)$

$$-9 = 3y - 6$$

$$-3y = 3$$

$$y = -1$$

Substitute the result for y in the first equation:

$$\frac{3}{x+1} + \frac{5}{-3} = 1$$

$$\frac{3}{x+1} = \frac{8}{3}$$

Cross-multiply and solve for x:

$$9 = 8x + 8$$

$$-8x = -1$$

$$x = \frac{1}{8}$$

SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 4.1–4.3 (showing handwritten computations and corrections)
- Choice of activity (labeled with the title of the activity):
  - Activity A: Integrated Review
  - Activity B: Citizen Math—Systems of Linear Equations in Society
  - Activity C: Which One Doesn't Fit? Solutions to Linear Systems
  - Activity D: NCTM Puzzle—Systems of Linear Equations

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson 4

Part 2: Systems of Equations

Lesson Introduction

Suggested time: 1.5 weeks

Lesson 4 continues with part 2. Refer to part 1 for learning objectives.

Exercise Sets

Read the following sections, and then complete the accompanying problem sets. Plan to complete a portion each day. If you have online access to MyMathLab, you can watch the instructional videos as well.

As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 4.4, "Solving Systems of Equations by Matrices" (231), and then complete the following problems in Exercise Set 4.4.
   - ☐ 1–23 odd
2. Read section 4.5, "Systems of Linear Inequalities" (235), and then complete the following problems in Exercise Set 4.5.
   - ☐ 1–23 odd
3. Read section 4.6, "Linear Programming" (241), and then complete the following problems in Exercise Set 4.6.
   - ☐ 1–25 EO odd
4. Optional: If you would like more practice, you have the option of completing the following, doing as many problems as needed.
   - ☐ Chapter 4 Review and Vocabulary Check (244)
   - ☐ Chapter 4 Standardized Test Practice (248)

ASSIGNMENT CHECKLIST

- ☐ Read sections 4.4–4.6 and complete the assigned problems.
- ☐ Complete the chapter 4 test.
- ☐ Complete the assessment test (if provided).
- ☐ Choose an activity to complete:

  Activity A: Fill in the Blank—Systems of Linear Inequalities

  Activity B: Math History—Matrices in Ancient China

  Activity C: Compare and Contrast Triple Systems

  Activity D: Real-World Application of Linear Programming

Chapter Test

1. In your textbook, complete the chapter 4 test on page 247. After completing the test, you or a supervising adult will grade it and mark the score at the top (for instance, 18/20). Then, review any mistakes and make necessary corrections.
2. For enrolled students: Complete the chapter 4 assessment test, if one has been provided.

Activities

Choose one of the following activities to complete.

- Activity A: Fill in the Blank—Systems of Linear Inequalities
- Activity B: Math History—Matrices in Ancient China
- Activity C: Compare and Contrast Triple Systems
- Activity D: Real-World Application of Linear Programming

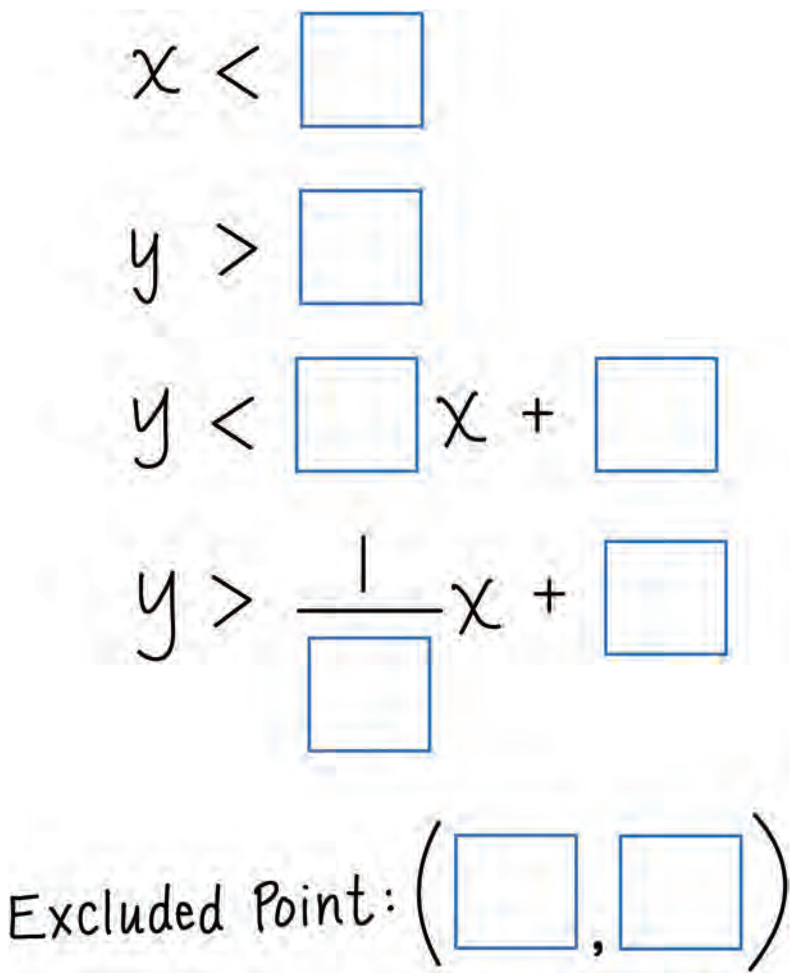
Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

Activity A: Fill in the Blank—Systems of Linear Inequalities

Use the integers –9 to 9, no more than once each, to fill in the blue squares so that the point (5, 6) is included within the solution region created by the following constraints and find an excluded value. Support your answer with a graph made by hand or use the Desmos online graphing calculator (www.desmos.com/calculator).



There are many possible solutions. Verify the student's work and that they did not use an integer more than once. Students should support their answer with a graph. One possible solution is shown below.

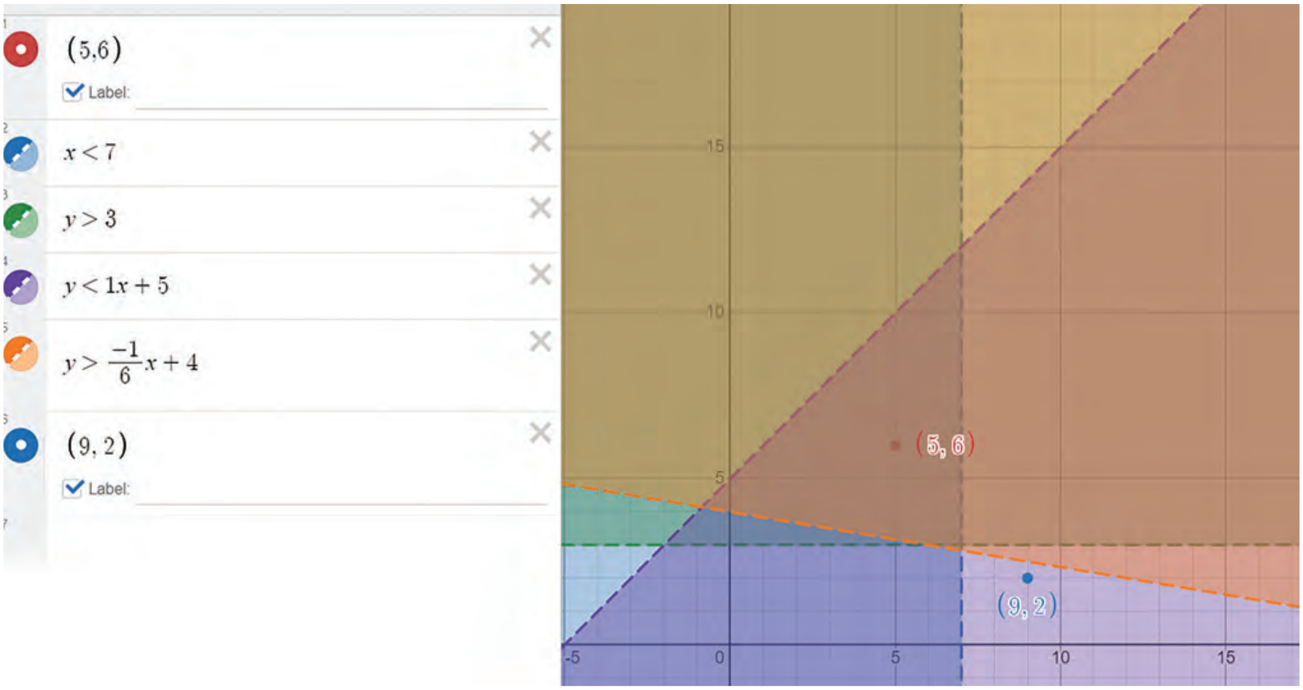
$x < 7$

$y > 3$

$y < 1x + 5$

$y > \frac{1}{-6x} + 4$

Excluded Point: (9, 2)

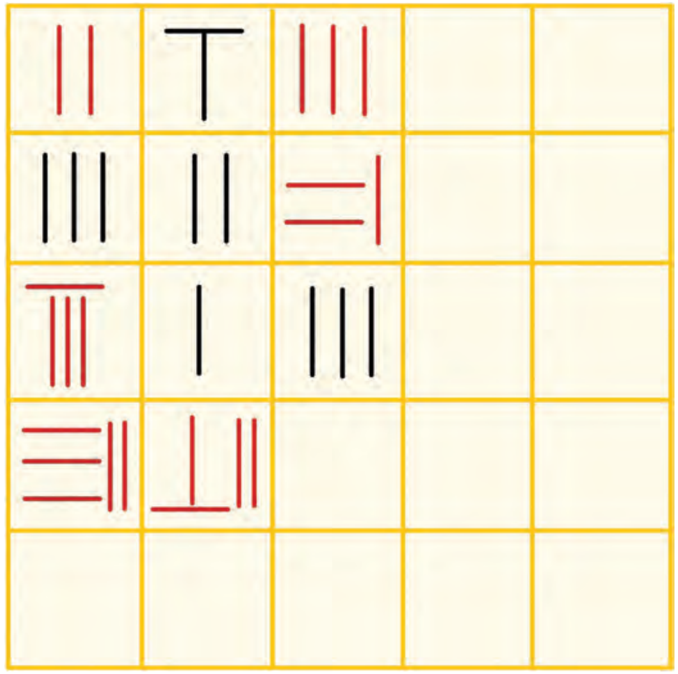


Activity B: Math History—Matrices in Ancient China

One of the most innovative aspects of ancient Chinese mathematics was the use of what we would equate today to matrices. Evidence of their use dates back to a text from the Han dynasty (200 BCE to 220 CE) called the *Jiu Zhang Suan Shu* (*Nine Chapters on the Mathematical Arts*). This text devoted an entire chapter to solutions of simultaneous equations called Fang cheng, which translates to "Method of Rectangular Arrays." This chapter contained 18 problems that involved solving simultaneous equations, with two to five unknown quantities, by placing them in a table and performing row operations identical to the methods we use to solve matrices today.

Chinese mathematicians used counting rods to represent coefficients of a system in a rectangular array, as shown below. Red rods represented positive numbers and black rods represented negative numbers. The array is read from right to left and top to bottom. The following array represents the given system. Observe the layout to see how the rods correspond to each equation and how they are oriented.

$$3x + 21y - 3z = 0$$
$$-6x - 2y - z = 62$$
$$2x - 3y + 8z = 32$$



Here is an example from the *Jiu Zhang Suan Shu* that shows how an ancient Chinese mathematician would have solved a system of equations.

> Five large containers and one small container have a total capacity of 3 *hu*; one large container and five small containers have a capacity of 2 *hu*. Find the capacities of one large container and one small container (1 *hu* = 10 *dou*).

Set up the information in an array as follows:

Large containers	1	5
Small containers	5	1
Total capacity	2	3

$$\begin{pmatrix} 1 & 5 \\ 5 & 1 \\ 2 & 3 \end{pmatrix}$$

Multiply each number in the first column by 5 and subtract the number in the second column. Replace the first column with these totals.

Large containers	0	5
Small containers	24	1
Total capacity	7	3

$$\begin{pmatrix} 0 & 5 \\ 24 & 1 \\ 7 & 3 \end{pmatrix}$$

Multiply each number in the second column by 24 and subtract the number in the first column. Replace the second column with these totals.

$$\begin{array}{l} \text{Large containers} \\ \text{Small containers} \\ \text{Total capacity} \end{array} \quad \begin{pmatrix} 0 & 120 \\ 24 & 0 \\ 7 & 65 \end{pmatrix}$$

This gives a solution of $\frac{7}{24}$ *hu* for a small container and $\frac{65}{120}$ $\left(\text{or } \frac{13}{21}\right)$ *hu* for a large container.

1. Show that this answer is correct using modern methods for solving matrices.

   Students should verify the solution using modern matrix procedures. There are multiple approaches that could be used to find the correct solution. One approach is shown below.

   Let x = large containers and y = small containers. This gives the equations:

   $5x + y = 3$ and $x + 5y = 2$, which can be arranged into a matrix, as follows.

$$\left[\begin{array}{cc|c} 5 & 1 & 3 \\ 1 & 5 & 2 \end{array}\right]$$

   Multiply the second row by –5 and add it to the first row. Replace the second row.

$$\left[\begin{array}{cc|c} 5 & 1 & 3 \\ 0 & -24 & -7 \end{array}\right]$$

   Multiply the first row by 24 and add to the second row. Replace the first row.

$$\left[\begin{array}{cc|c} 120 & 0 & 65 \\ 0 & -24 & -7 \end{array}\right]$$

   This allows us to solve for y by setting up the equation $-24y = -7$, which gives $y = \frac{7}{24}$, and the equation $120x = 65$, which gives $x = \frac{65}{120}$, which reduces to $\frac{13}{21}$. Therefore, the solution is $\left(\frac{13}{21}, \frac{7}{24}\right)$ and verifies the example.

2. Solve the following problem from the *Jiu Zhang Suan Shu* using the ancient Chinese method. Annotate your work.

   The yield of 2 sheaves of superior grain, 3 sheaves of medium grain, and 4 sheaves of inferior grain is each less than 1 *dou*. But if 1 sheaf of medium grain is added to the superior grain or if 1 sheaf of inferior grain is added to the medium, or if 1 sheaf of superior grain is added to the inferior, then in each case the yield is exactly 1 *dou*. What is the yield of one sheaf of each grade of grain?

   In modern-day notation, we would let x be the yield from 1 sheaf of superior grain, y be the yield from 1 sheaf of medium grain, and z be the yield from 1 sheaf of inferior grain. The first statement tells us that $2x \le 1$, $3y \le 1$, and $4z \le 1$. We can use this information to set up three equations. For example, if 1 sheaf of medium grain is added to the 2 sheaves of superior grain, the yield is exactly 1, which is represented by the equation $2x + y = 1$.

   Set up the other two equations and arrange them in a vertical matrix. Solve the problem and explain each of your steps.

   Students should first set up the other two equations in addition to the one given.

   $2x + y = 1$ (given)

   $3y + z = 1$

   $4z + x = 1$

   They should then arrange them into vertical columns in a four-by-three matrix, as follows, and solve, annotating all their work. There are multiple approaches that could be used to solve the problem. One solution is below.

Superior	1	0	2
Medium	0	3	1
Inferior	4	1	0
Total yield	1	1	1

   Multiply each number in the first column by 2 and subtract each number in the third column. Replace the first column with these totals.

Superior	0	0	2
Medium	−1	3	1
Inferior	8	1	0
Total yield	1	1	1

Multiply each number in the first column by 3 and add each number in the middle column. Replace the first column with these totals.

Superior	0	0	2
Medium	0	3	1
Inferior	25	1	0
Total yield	4	1	1

This has successfully cleared the first column and made it possible to solve for the inferior variable. 25 sheaves of inferior grain yield 4 *dou*, so 1 sheaf of inferior grain yields $\frac{4}{25}$ *dou* $\left(z = \frac{4}{25}\right)$.

Substituting this value into the middle column, we see that 3 sheaves of medium grain plus $\frac{4}{25}$ of inferior grain yield 1 *dou*. Subtracting $\frac{4}{25}$ from 1 and dividing by 3 show that 1 sheaf of medium grain yields $\frac{7}{25}$ *dou* $\left(y = \frac{7}{25}\right)$.

Substituting into the third column, we see that 2 sheaves of superior grain plus $\frac{7}{25}$ of medium grain yield 1 *dou*. Subtracting $\frac{7}{25}$ from 1 and dividing by 4 shows that 1 sheaf of superior grain yields $\frac{9}{25}$ *dou* $\left(x = \frac{9}{25}\right)$.

3. What was your favorite part of this activity?

   Answers will vary. Students should describe their favorite part of the activity. For instance, they might have enjoyed the challenge of working in a vertical orientation.

4. What do you still wonder about?

   Students should pose additional questions related to these specific problems or ancient Chinese mathematics in general.

Activity C: Compare and Contrast Triple Systems

Two students were given the following problem to solve, and they included all their work. The students each used different approaches to solve the problem.

$$\frac{1}{3}x - \frac{1}{4}y + z = 3$$
$$2x - y + z = -2$$
$$x + 3y - 4z = -19$$

Student A's approach:

① $\frac{1}{3}x - \frac{1}{4}y + z = 3$

② $2x - y + z = -2$

③ $x + 3y - 4z = -19$

multiply ① through by LCD of 12:

$12\left(\frac{1}{3}x - \frac{1}{4}y + z = 3\right)12$

① $4x - 3y + 12z = 36$

eliminate y by adding ① and ③ together:

① $4x - 3y + 12z = 36$

③ $+\ x + 3y - 4z = -19$

④ $5x + 8z = 17$

eliminate y by adding 3·② and ③ together:

3② $6x - 3y + 3z = -6$

③ $+\ x + 3y - 4z = -19$

⑤ $7x - z = -25$

eliminate z by adding ④ and 8·⑤ together:

④ $5x + 8z = 17$

8·⑤ $+\ 56x - 8z = -200$

$61x = -193$

$x = -3$

plug x into ④ to find z:

$5(-3) + 8z = 17$

$-15 + 8z = 17$

$8z = 32$

$z = 4$

plug x and z into ② to find y:

$2(-3) - y + 4 = -2$

$-6 - y + 4 = -2$

$-2 - y = -2$

$-y = 0$

$y = 0$

Solution: (−3, 0, 4)

Student B's approach:

$$\frac{1}{3}x - \frac{1}{4}y + z = 3$$
$$2x - y + z = -2$$
$$x + 3y - 4z = -19$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & -19 \\ 2 & -1 & 1 & -2 \\ \frac{1}{3} & -\frac{1}{4} & 1 & 3 \end{array}\right] \quad -3R_3 + R_1$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & -19 \\ 2 & -1 & 1 & -2 \\ 0 & \frac{15}{4} & -7 & -28 \end{array}\right] \quad -\frac{1}{2}R_2 + R_1$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & -19 \\ 0 & \frac{7}{2} & -\frac{9}{2} & -18 \\ 0 & \frac{15}{4} & -7 & -28 \end{array}\right] \quad -\frac{9}{2}R_3 + 7R_2$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & -19 \\ 0 & \frac{7}{2} & -\frac{9}{2} & -18 \\ 0 & \frac{61}{8} & 0 & 0 \end{array}\right]$$

Solve:

$$\frac{61}{8}y = 0$$
$$y = 0$$

$$\frac{7}{2}(0) - \frac{9}{2}z = -18$$
$$-\frac{2}{9} \cdot -\frac{9}{2}z = -18 \cdot -\frac{2}{9}$$
$$z = 4$$

$$x + 3(0) - 4(4) = -19$$
$$x - 16 = -19$$
$$x = -3$$

Solution: $(-3, 0, 4)$

1. Compare and contrast the two methods. How are they the same? How are they different?

   Answers will vary. Students should thoroughly explore similarities and differences between the two methods. Sample answers are below.

   Similarities: Both students annotated their work, so it was clear what steps they were following; both students were able to get one equation with one variable to solve for a single variable; both students used substitution to find the remaining variables; both students wrote their solution as a triple ordered pair, etc.

   Differences: Student A used elimination to solve, and Student B used matrices; Student A eliminated the fractions at the beginning of their work, and Student B kept fractions throughout the problem; Student A solved for *z* first, and Student B solved for *y* first; Student B reordered the equations to start the matrix with a 1 in the top left corner, etc.

2. What are the benefits and challenges of each method? Which method did you prefer and why?

   Students should discuss the benefits and challenges of each method and explain which method they prefer and why. Possible benefits to Student A's method are eliminating the fractions to make the system easier to work with, choosing the first variable to eliminate by what would require the least amount of multiplication, etc. Possible benefits to Student B's method are keeping the equations as is to avoid any mistakes with multiplying through at the beginning, only having to work with the matrix until the value of one variable can be determined and then working backward with substitution to solve the rest, etc. Students should explain their preference for one method (or parts of each method).

3. Solve this problem a third way using the method of your choice. Show all your work.

   There are a number of other ways students could solve this problem. They could use the elimination method, keeping the fractions; they could use the elimination method and start by eliminating x or y instead of z; they could use the matrix method with fractions eliminated; they could use the matrix method with the equations ordered differently at the beginning; they could use the matrix method and start by solving for x or z instead of y; they could use the matrix method and complete it all the way through so all variables are solved for within the matrix, etc. Students could also solve this system using substitution.

   Verify the student's work and that they approached the problem with at least one significant difference from the provided examples.

Activity D: Real-World Application of Linear Programming

Complete the following activity from your textbook.

On June 24, 1948, the former Soviet Union blocked all land and water routes through East Germany to Berlin. A gigantic airlift was organized using American and British planes to bring food, clothing, and other supplies to the more than 2 million people in West Berlin. The cargo capacity was 30,000 cubic feet for an American plane and 20,000 cubic feet for a British plane. To break the Soviet blockade, the Western Allies had to maximize cargo capacity, but were subject to the following restrictions:

- No more than 44 planes could be used.
- The larger American planes required 16 personnel per flight, double that of the requirement for the British planes. The total number of personnel available could not exceed 512.
- The cost of an American flight was \$9,000 and the cost of a British flight was \$5,000. Total weekly costs could not exceed \$300,000.

Find the number of American planes and the number of British planes that were used to maximize cargo capacity.

1. Write an objective function and system of constraints to model this scenario.

   Students should define variables: Let x = American and y = British (or vice versa)

   Objective Function: $z = 30{,}000x + 20{,}000y$

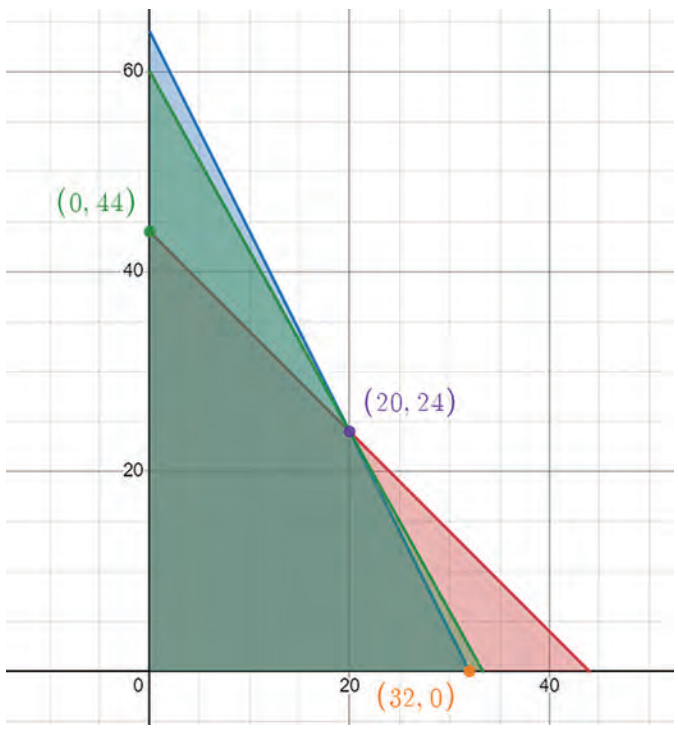
Constraints: $x + y \leq 44$

$16x + 8y \leq 512$

$9{,}000x + 5{,}000y \leq 300{,}000$

2. Use the Desmos online graphing calculator (www.desmos.com/calculator) to graph the system. Make sure to restrict the graph appropriately. Label all vertex points. Include a screenshot of your graph with your submission.

   Students should correctly graph the system of constraints, restrict the domain and range to Quadrant 1, and label all vertex points as shown below:



3. Find the number of American and British planes used to maximize cargo capacity.

   Students should test all vertex points in the objective function to see which one maximizes the cargo capacity.

   (0, 44) $z = 30{,}000(0) + 20{,}000(44) = 880{,}000$

   (32, 0) $z = 30{,}000(32) + 20{,}000(0) = 960{,}000$

   (20, 24) $z = 30{,}000(20) + 20{,}000(24) = 1{,}080{,}000$

   Cargo capacity is maximized when there are 20 American planes and 24 British planes.

SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 4.4–4.6 (showing handwritten computations and corrections)
- Chapter 4 test
- Chapter 4 assessment test (if one has been provided)
- Choice of activity (labeled with the title of the activity):
  - Activity A: Fill in the Blank—Systems of Linear Inequalities
  - Activity B: Math History—Matrices in Ancient China
  - Activity C: Compare and Contrast Triple Systems
  - Activity D: Real-World Application of Linear Programming

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.