

Geometry

High School Teacher Edition



Oak Meadow

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Teacher Edition



Oak Meadow

Oak Meadow, Inc.

Post Office Box 615

Putney, Vermont 05346

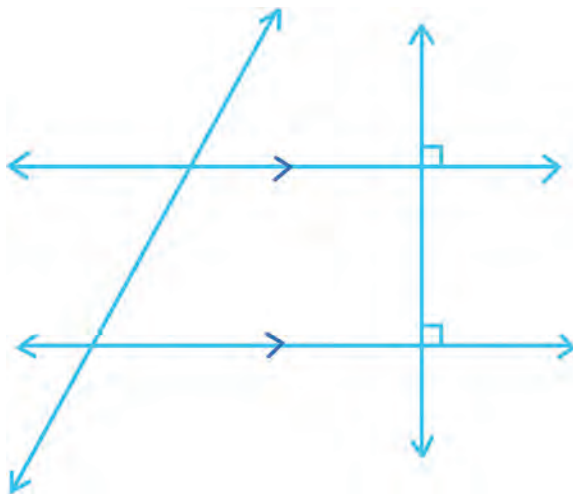
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Lesson

3

Part 1: Parallel and Perpendicular Lines

Exploratory Activity



What are three (or more) things you recognize in this picture?

Most of the exploratory activities are open-ended questions meant to probe thinking and are not necessarily designed to have a unique solution.

Students might recognize lines, perpendicular lines, right angles, vertical angles, intersecting lines, linear pairs, etc.

What are three (or more) things you have questions about in this picture?

Students might question what the two dark blue arrows mean, what other angle relationships are present, if the two horizontal lines are parallel, etc.

Lesson Introduction

Suggested time: 1.5 weeks

Parallel lines occur all around us. You could look around the room right now and spot several examples: opposite edges of tables, the sides of picture frames, the edges of windows. Parallel lines have

ASSIGNMENT CHECKLIST

- ☐ Complete the exploratory activity.
- ☐ Read sections 3.1–3.4 and complete the assigned problems.
- ☐ Choose an activity to complete:

Activity A: Integrated Review

Activity B: Error Analysis of Proof Writing

Activity C: Fill in the Blank—Parallel Lines and Transversals

the special property of never intersecting and, therefore, create a lot of unique relationships between them, as we will see in this lesson.

Learning Objectives

Use the checklist below to track how your skills are developing over time, and identify skills that need more work as you progress through part 1 and part 2 of this lesson.

Skills	Notes
Identify parallel, perpendicular, and skew lines and planes	
Identify the angles formed by parallel lines and a transversal and their relationship to each other	
Use and prove theorems about parallel and perpendicular lines	
Construct parallel and perpendicular lines with a straightedge and compass	
Graph equations of lines on the xy -coordinate plane, including horizontal and vertical lines	
Find the slope and y -intercept of a line	
Write an equation of a line with specific conditions, including a line parallel or perpendicular to a given line	
Determine if two lines are parallel or perpendicular given their equations	

Exercise Sets

Read and/or watch instructional videos for the following sections, and complete the accompanying problem sets. Plan to complete a portion each day. As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 3.1, “Lines and Angles” (106), and then complete the following problems in Exercise Set 3.1.

□ 1–47 odd

2. Read section 3.2, “Proving Lines Are Parallel” (114), and then complete the following problems in Exercise Set 3.2.
 - 1–37 odd
3. Read section 3.3, “Parallel Lines and Angles Formed by Transversals” (120), and then complete the following problems in Exercise Set 3.3.
 - 1–45 odd
 - 47–50 all (proof)
 - Extension 51–57 odd
4. Read section 3.4, “Proving Theorems About Parallel and Perpendicular Lines” (125), and then complete the following problems in Exercise Set 3.4.
 - 1–27 odd
 - Extension 29

Activities

Choose one of the following activities to complete.

- Activity A: Integrated Review
- Activity B: Error Analysis of Proof Writing
- Activity C: Fill in the Blank—Parallel Lines and Transversals

Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	

	Notes
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

Activity A: Integrated Review

In your textbook, complete the integrated review on page 152, problems 1–17 odd, 35, and 37. Show all your work. Check your answers, and make corrections to any problems you missed.

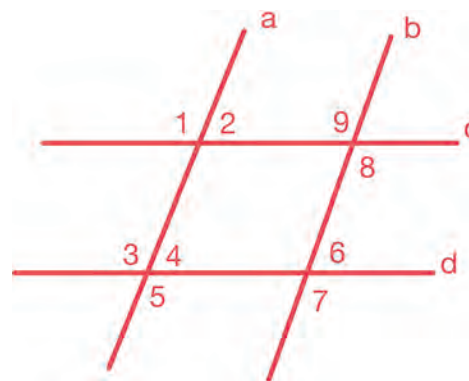
Activity B: Error Analysis of Proof Writing

This is a great opportunity to strengthen your proof writing skills and get constructive feedback before moving on to more proofs in the next lesson.

Examine the following proof and identify the error(s).

Given: $a \parallel b$, $\angle 5 \cong \angle 9$

Prove: $\angle 5$ is supplementary to $\angle 2$



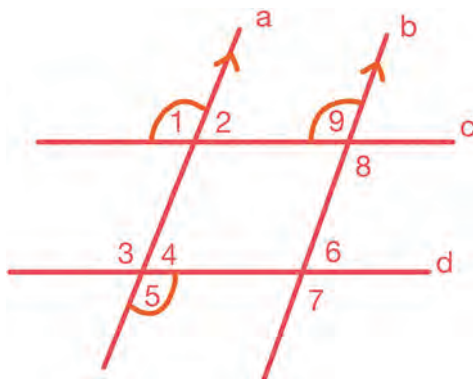
STATEMENTS	REASONS
1. $a \parallel b$, $\angle 5 \cong \angle 9$	1. Given
2. $\angle 9 \cong \angle 7$	2. If \parallel lines, then alt. ext. angles \cong
3. $\angle 5 \cong \angle 7$	3. Transitive property
4. $\angle 5$ is supplementary to $\angle 4$	4. Linear pairs are supplementary
5. $\angle 4 \cong \angle 2$	5. If \parallel lines, then corresponding angles \cong
6. $\angle 5$ is supplementary to $\angle 2$	6. Transitive property

1. What mistake(s) did this student make?

Students should identify that the main mistake in this proof is using the wrong pair of lines in step 2. The student has incorrectly interpreted $a \parallel b$ and instead used $c \parallel d$ and b as a transversal to conclude step 2. Students should also recognize that the angles in step 3 are actually congruent because they are corresponding angles on a transversal for $a \parallel b$.

2. Write the proof correctly.

Verify that students correctly prove the objective, ideally in the shortest route possible. One solution is:



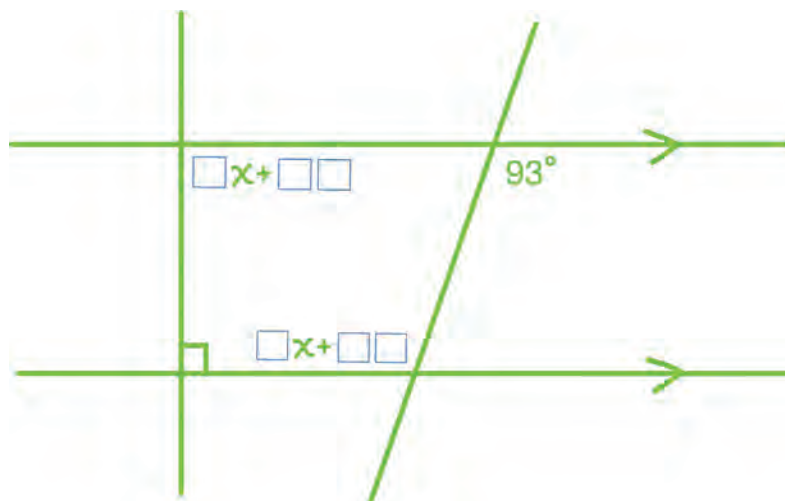
Given: $a \parallel b$, $\angle 5 \cong \angle 9$

Prove: $\angle 5$ is supplementary to $\angle 2$

STATEMENTS	REASONS
1. $a \parallel b$, $\angle 5 \cong \angle 9$	1. Given
2. $\angle 9 \cong \angle 1$	2. If \parallel lines, then corresponding angles \cong
3. $\angle 5 \cong \angle 1$	3. Transitive property
4. $c \parallel d$	4. If alt. ext. angles \cong , then \parallel lines
5. $\angle 5$ is supplementary to $\angle 2$	5. If \parallel lines, then same side ext. angles supplementary

Activity C: Fill in the Blank—Parallel Lines and Transversals

Fill in the blue squares with the digits 0 through 9, using each number only once, so that x is a whole number that satisfies both missing angles. Show all your work demonstrating how you obtained your answer.



Verify the student's work, making sure they used each digit only once and that x is a whole number. There are many possible solutions. Two examples are below.

$$5x + 10 = 90$$

$$4x + 29 = 93$$

$$\text{So } x = 16$$

$$3x + 57 = 90$$

$$7x + 16 = 93$$

$$\text{So } x = 11$$

Challenge yourself! Choose a different value for the given angle, 93° , and find at least two ways to solve the problem.

Verify the student's work for the extension. They should set a new value to replace 93 and find at least two different ways to fill in the blanks according to the criteria of the original problem.

SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 3.1–3.4 (showing handwritten computations and corrections)
- Choice of activity (labeled with the title of the activity):
 - Activity A: Integrated Review
 - Activity B: Error Analysis of Proof Writing
 - Activity C: Fill in the Blank—Parallel Lines and Transversals

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson

3

Part 2: Parallel and Perpendicular Lines

Lesson Introduction

Suggested time: 1.5 weeks

Lesson 3 continues with part 2. Refer to part 1 for learning objectives.

Exercise Sets

Read and/or watch instructional videos for the following sections, and complete the accompanying problem sets. Plan to complete a portion each day. As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 3.5, “Constructions: Parallel and Perpendicular Lines” (130), and then complete the following problems in Exercise Set 3.5.
 - 1–19 odd
 - Extension 25–35 odd
2. Read section 3.6, “Coordinate Geometry: The Slope of a Line” (137), and then complete the following problems in Exercise Set 3.6.
 - 1–33 EO odd
3. Read section 3.7, “Coordinate Geometry: Equations of Lines” (147), and then complete the following problems in Exercise Set 3.7.
 - 1–45 EO odd
 - read page 150

ASSIGNMENT CHECKLIST

- Read sections 3.5–3.7 and complete the assigned problems.
- Complete the chapter 3 test.
- Choose an activity to complete:

Activity A: Explore with Technology
Constructing Parallel and Perpendicular Lines

Activity B: Which One Doesn't Fit? Equations of Lines

Activity C: Get Creative with Parallel and Perpendicular Lines

Activity D: Real-World Application of Parallel and Perpendicular Lines

4. Optional: If you would like more practice, you have the option of completing the following, doing as many problems as needed.
- Chapter 3 Review and Vocabulary Check (152)
 - Chapter 3 Standardized Test Practice (154)

Chapter Test

In your textbook, complete the chapter 3 test on page 153. After completing the test, you or a supervising adult will grade it and mark the score at the top (for instance, 18/20). Then, review any mistakes and make necessary corrections.

Activities

Choose one of the following activities to complete.

- Activity A: Explore with Technology Constructing Parallel and Perpendicular Lines
- Activity B: Which One Doesn't Fit? Equations of Lines
- Activity C: Get Creative with Parallel and Perpendicular Lines
- Activity D: Real-World Application of Parallel and Perpendicular Lines

Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	

	Notes
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

Activity A: Explore with Technology Constructing Parallel and Perpendicular Lines

Learn to construct parallel and perpendicular lines with technology. Complete the constructions in the following GeoGebra activity. (All online resources can be accessed at oakmeadow.com/curriculum-links.)

“Constructing Parallel and Perpendicular Lines”

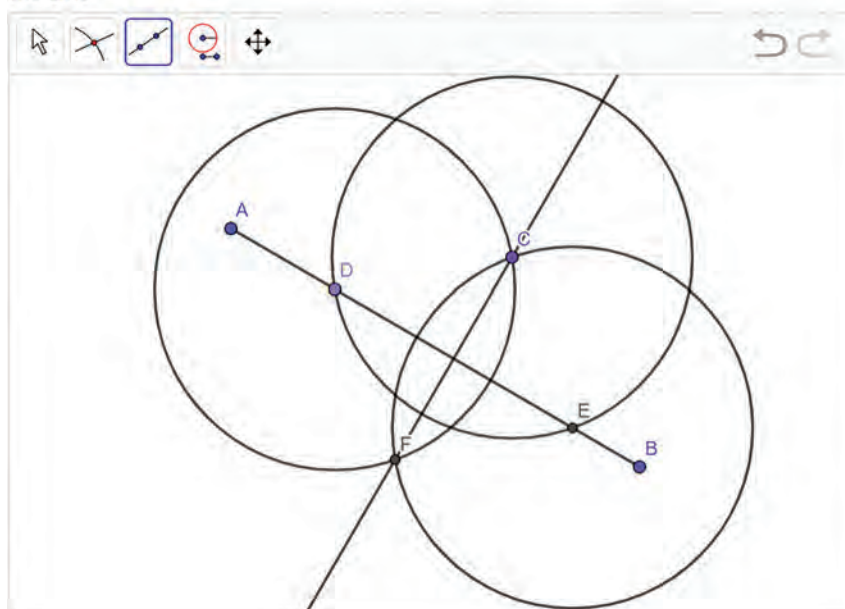
1. Which method did you prefer for these constructions (by hand or with technology) and why?

Students may prefer the tactile nature of doing them by hand, the simplicity of the final product, not having to make full circles, etc. They may prefer using technology because it can be more precise, it can be an easier tool to use, etc.

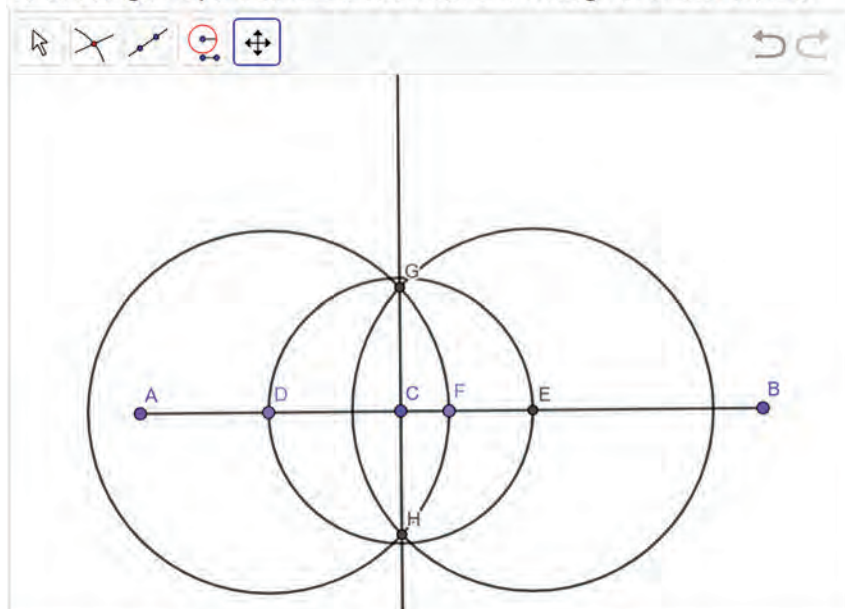
2. Include a screenshot of each of your constructions with your submission.

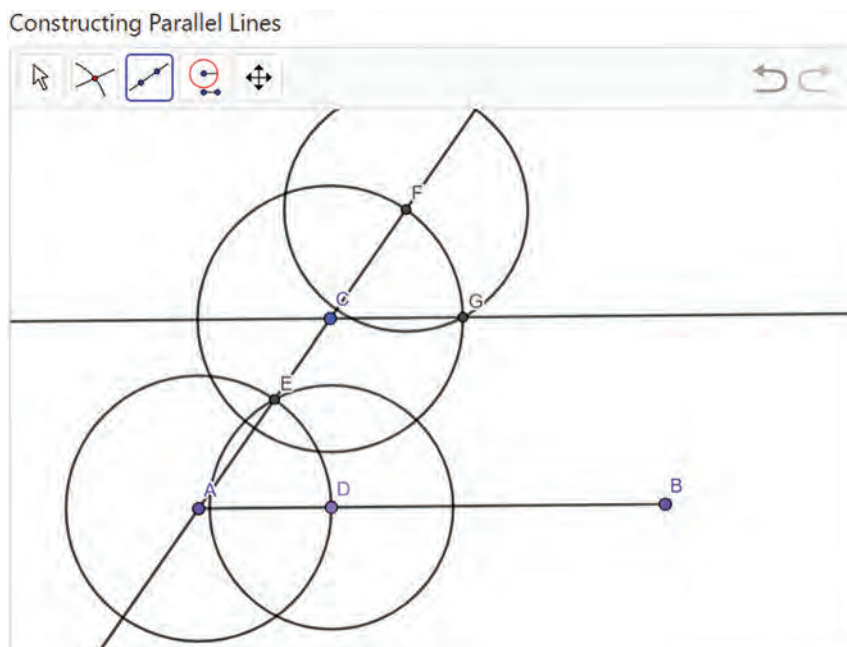
Students should include screenshots of their work to demonstrate proper steps and outcomes in their constructions. Verify the student's work. Correct work may have slight variations, but examples of constructions are below.

Constructing a Perpendicular Line to a Given Line and a Given Point Not on the Line



Constructing a Perpendicular Line to a Given Line Through a Point on the Line





Activity B: Which One Doesn't Fit? Equations of Lines

<p>A</p> $y = 7$ $x = -3$	<p>B</p>	<p>C</p> $y = -\frac{2}{3}x + 4$ $6x + 9y = 2$	<p>D</p> $(3, 0)$ $(2, 4)$ $(6, 5)$ $(7, 1)$
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Which one doesn't fit?

Think beyond the presentation to what this information represents. Come up with two different ways to answer this question. Explain your reasoning and include any work done to arrive at your conclusion(s).

Students should justify how two different options do not fit. Possible solutions are below.

A does not fit because it is the only pair of lines that is only perpendicular. They are also the only pair of lines that are horizontal and vertical.

B does not fit because it is the only pair of lines that is neither parallel nor perpendicular. Students should justify this with some evidence of slope calculations or linear equation writing. The two lines are $y = \frac{5}{3x} + 5$ and $y = \frac{4}{5x} + 1$.

C does not fit because it is the only pair of lines that is only parallel. Students should justify this with some evidence of comparing the slopes of the two given lines by manipulating the equations to the same form. The two lines are $y = \frac{-2}{3x} + 4$ and $y = \frac{-2}{3x} + \frac{2}{9}$.

D does not fit because it is the only option that can create parallel or perpendicular lines depending on how the points are connected. Students should justify this with some evidence of graphing the points and playing around with possible lines, and/or parallel and perpendicular lines should be verified with slope calculations or linear equation writing. For example, if the points (3, 0) and (7, 1) are connected and the points (2, 4) and (6, 5) are connected, the lines are parallel with a slope of $\frac{1}{4}$. If the points (3, 0) and (2, 4) are connected and the points (6, 5) and (7, 1) are connected, the lines are parallel with a slope of -4 . If the points (3, 0) and (6, 5) are connected and the points (2, 4) and (7, 1) are connected, the lines are perpendicular with slopes of $\frac{5}{3}$ and $-\frac{3}{5}$, respectively.

Activity C: Get Creative with Parallel and Perpendicular Lines

Use Desmos to create a digital art piece using parallel and perpendicular lines. Your picture can be as simple as the example found below, or as intricate as you like. (All online resources can be accessed at oakmeadow.com/curriculum-links.)

“Algebra 1 Chapter 3 Part 2 Option 4 Example”

Use the example to help you figure out how to restrict the domain or range of your line to cut it off where you would like. Simply type in the equation of the line and use brackets and inequality symbols to restrict the domain or range, or check out the following resource:

“Getting Started: Inequities and Restrictions”

Your picture must contain the following:

- at least 10 lines total (You may include more if you want!)
 - at least 3 pairs of parallel lines
 - at least 3 pairs of perpendicular lines
1. Save your graph and submit the link to your teacher so they can examine your picture and all your equations (or take a screenshot of your work as long as you include the whole picture and all equations).

Answers will vary. See the link above for an example. Students must meet the minimum line requirements stated and should have the properly restricted domain and range for each of their equations.

2. What is your favorite part of your creation?

Students should describe their favorite aspect of their creation.

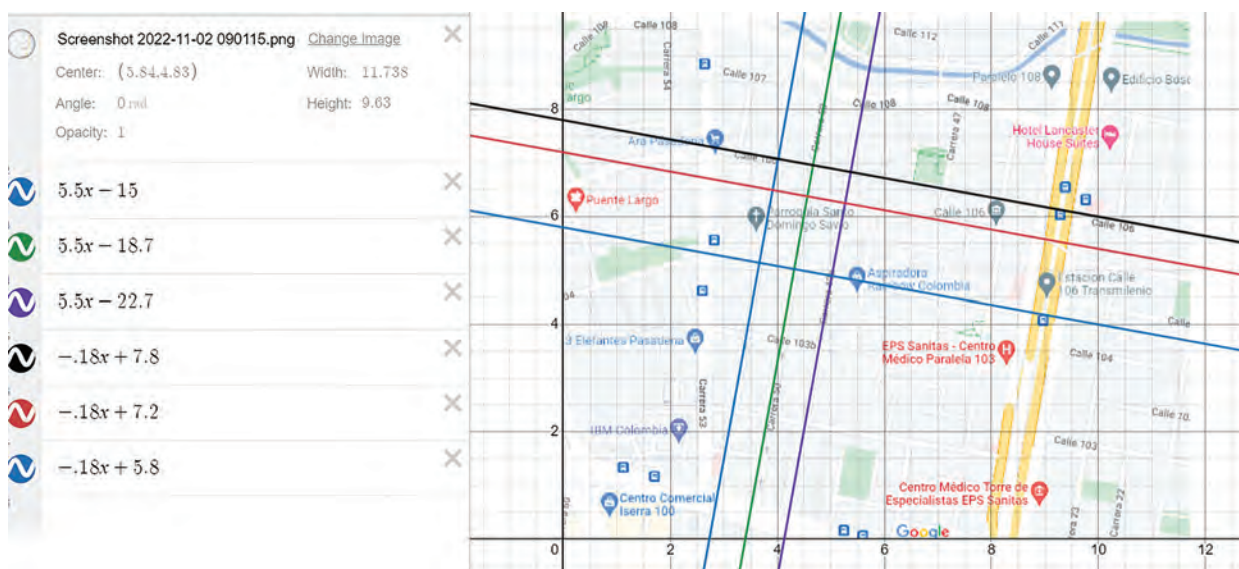
Activity D: Real-World Application of Parallel and Perpendicular Lines

We can find parallel and perpendicular lines all around us. One of the most common places we might see them is in the layout of streets in towns and cities. Go to Google Maps (google.com/maps) and find

a city near you, one you have visited, one you would like to visit someday, or one you are curious about anywhere in the world. Examine the city streets and take a screenshot of a section where you see both parallel and perpendicular lines.

1. Upload your screenshot into [desmos.com/calculator](https://www.desmos.com/calculator) so that the coordinate plane is overlaid on top of the image. (Select the “+” icon in the upper left corner to add an image. You can adjust the center of the image once it is uploaded by editing the center point under the image information or clicking on the center point of the image and dragging it to the location you want.)
2. Trace at least three parallel lines and three perpendicular lines.
3. Take a screenshot of each of your lines, including the equations.

Answers will vary. Verify the student’s work. They should have traced at least three parallel and three perpendicular city streets and included a screenshot that displays their equations. Example:



You can find the answer key at oakmeadow.com/answer-keys.

SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 3.5–3.7 (showing handwritten computations and corrections)
- Chapter 3 test
- Choice of activity (labeled with the title of the activity):
 - Activity A: Explore with Technology Constructing Parallel and Perpendicular Lines

- Activity B: Which One Doesn't Fit? Equations of Lines
- Activity C: Get Creative with Parallel and Perpendicular Lines
- Activity D: Real-World Application of Parallel and Perpendicular Lines

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson

4

Part 1: Triangles and Congruence

Exploratory Activity

$$\triangle PIG \cong \triangle CAT$$

$$\angle I \cong \angle A$$

$$\angle GPI \cong \angle TCA$$

$$PI \cong CA$$

$$PG \cong CT$$

What do you notice?

Students might notice that angles and sides that are in the same place in the triangle are congruent to each other (the middle letters represent congruent angles).

What do you wonder?

Students might wonder if angle T is congruent to angle G and if IG is congruent to AT , how this looks in a diagram, if corresponding parts are congruent in all triangles, etc.

Lesson Introduction

Suggested time: 1.5 weeks

Triangles exist all around us and have very important characteristics. Did you know that the triangle is the strongest shape? This is why it appears so often in architectural designs. In this lesson, you will be exploring **congruent triangles**—what, specifically, makes two triangles congruent, what parts of congruent triangles are congruent to each other, and how to prove two triangles are congruent. You will also investigate special types of triangles and the relationship between their angles and sides, and have an opportunity to expand your proof writing skills.

ASSIGNMENT CHECKLIST

- ☐ Complete the exploratory activity.
- ☐ Read sections 4.1–4.3 and complete the assigned problems.
- ☐ Choose an activity to complete:

Activity A: Integrated Review

Activity B: Which One Doesn't Fit? Corresponding Parts of Triangles

Activity C: Fill in the Blank—Triangle Sum and Exterior Angles

Activity D: Puzzle—Angles in a Triangle

Learning Objectives

Use the checklist below to track how your skills are developing over time, and identify skills that need more work as you progress through part 1 and part 2 of this lesson.

Skills	Notes
Classify triangles by their sides and angles	
Find angle measures in triangles using the triangle sum theorem and properties of isosceles, equilateral, and right triangles	
Identify corresponding parts in congruent triangles	
Identify appropriate assumptions that can be made from a diagram and identify congruent triangles by SSS, SAS, ASA, AAS, and HL	
Write a two-column proof involving overlapping triangles and use Corresponding Parts of Congruent Triangles Are Congruent (CPOCTAC) to prove parts are congruent or to reach the given objective	

Exercise Sets

Read and/or watch instructional videos for the following sections, and complete the accompanying problem sets. Plan to complete a portion each day. As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

- Read section 4.1, “Types of Triangles” (162), and then complete the following problems in Exercise Set 4.1.
 - ☐ 1–51 odd
- Read section 4.2, “Congruent Figures” (168), and then complete the following problems in Exercise Set 4.2.
 - ☐ 1–37 odd
 - ☐ 39–42 all (proof)
 - ☐ 43
 - ☐ Extension 48

3. Read section 4.3, “Congruent Triangles by SSS and SAS” (174), and then complete the following problems in Exercise Set 4.3.

- 1–33 odd
- Extension 35–41 odd

Activities

Choose one of the following activities to complete.

- Activity A: Integrated Review
- Activity B: Which One Doesn’t Fit? Corresponding Parts of Triangles
- Activity C: Fill in the Blank—Triangle Sum and Exterior Angles
- Activity D: Puzzle—Angles in a Triangle

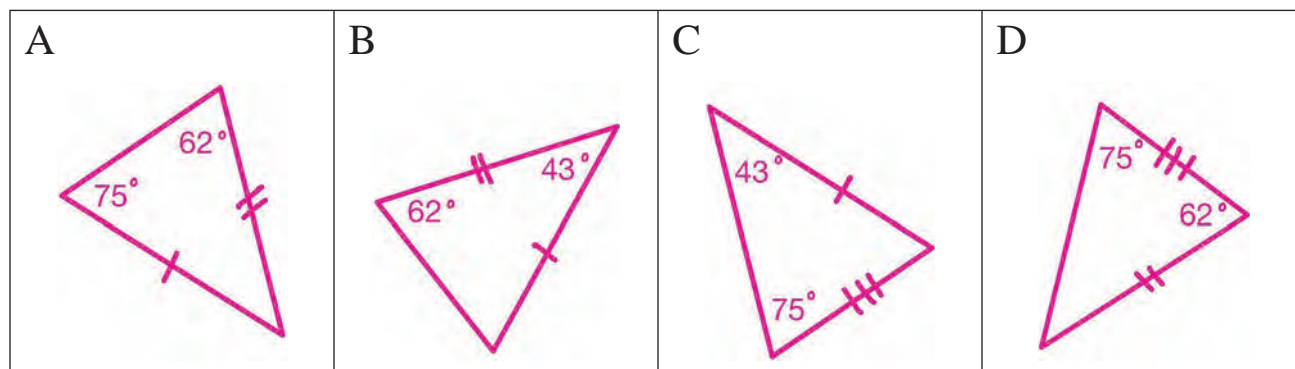
Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

Activity A: Integrated Review

In your textbook, complete the integrated review on page 199, problems 1–19 odd, 41, and 43. Show all your work. Check your answers, and make corrections to any problems you missed.

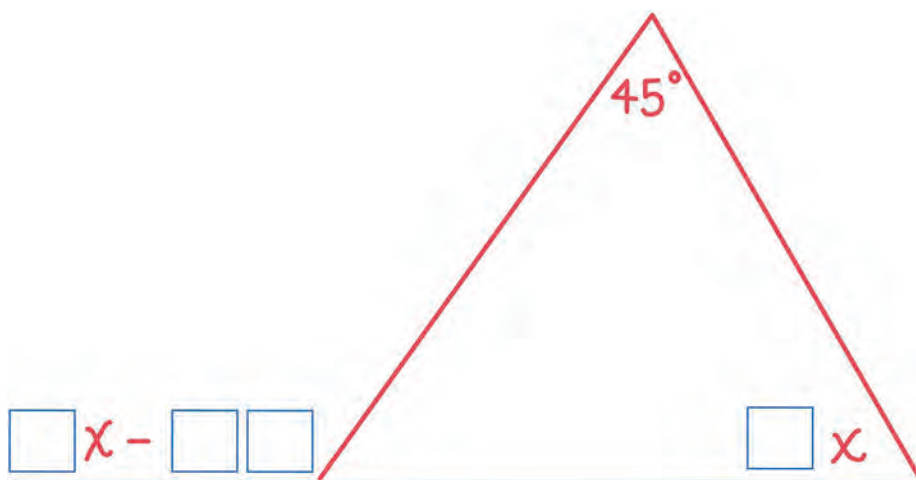
Activity B: Which One Doesn't Fit? Corresponding Parts of Triangles

Given that these images represent the same triangle, which one doesn't fit? Explain your answer.

Students should identify that option C does not fit and explain how they know. Option C is the only one where corresponding angles and corresponding sides of the triangle do not match up. Filling in the missing angle from each triangle shows that the side with one tick mark should appear between the 75- and 43-degree angles, the side with two tick marks appears between the 62- and 43-degree angles, and the side with three tick marks appears between the 75- and 62-degree angles. Option C does not meet these criteria, and therefore does not fit with the rest.

Activity C: Fill in the Blank—Triangle Sum and Exterior Angles

Fill in the blue squares with the digits 1 through 9, using each number only once, so that the value for x is a whole number. Show all your work demonstrating how you obtained your answer.



Students should show all their work and include calculations involving the exterior angle theorem. Verify the student's work, making sure they used each digit only once and that x is a whole number. There are many possible solutions; three are below.

$$9x - 15 = 45 + 4x \text{ where } x = 12$$

$$6x - 21 = 45 + 3x \text{ where } x = 11$$

$$5x - 48 = 45 + 2x \text{ where } x = 31$$

Challenge yourself! (optional) Choose a whole number value for x . Then, find all the ways to make a problem with that solution, again using the digits 1 through 9 only once each.

This activity is optional. Verify the student's work. They should set a whole number value for x and find all the ways to fill in the problem. There are many possible solutions. One example is below.

Set $x = 16$

Students should try to work backwards by finding all multiples of 16 and divisors that work within the constraints of the problem (in this case 4 through 8). Then based on the digits needed to fill in the two-digit number that adds to 45 to create the multiple of 16, they can find all the ways to fill in the coefficients to the x -values to create the needed divisors so that each digit is used only once.

$$7x - 19 = 45 + 3x$$

$$8x - 19 = 45 + 4x \quad \text{all create } 4x = 64$$

$$6x - 19 = 45 + 2x$$

$$9x - 35 = 45 + 4x$$

$$7x - 35 = 45 + 2x \quad \text{all create } 5x = 80$$

$$6x - 35 = 45 + 1x$$

$$9x - 51 = 45 + 3x \quad \text{all create } 6x = 96$$

$$8x - 51 = 45 + 2x$$

$$9x - 67 = 45 + 2x \quad \text{all create } 7x = 112$$

$$8x - 67 = 45 + 1x$$

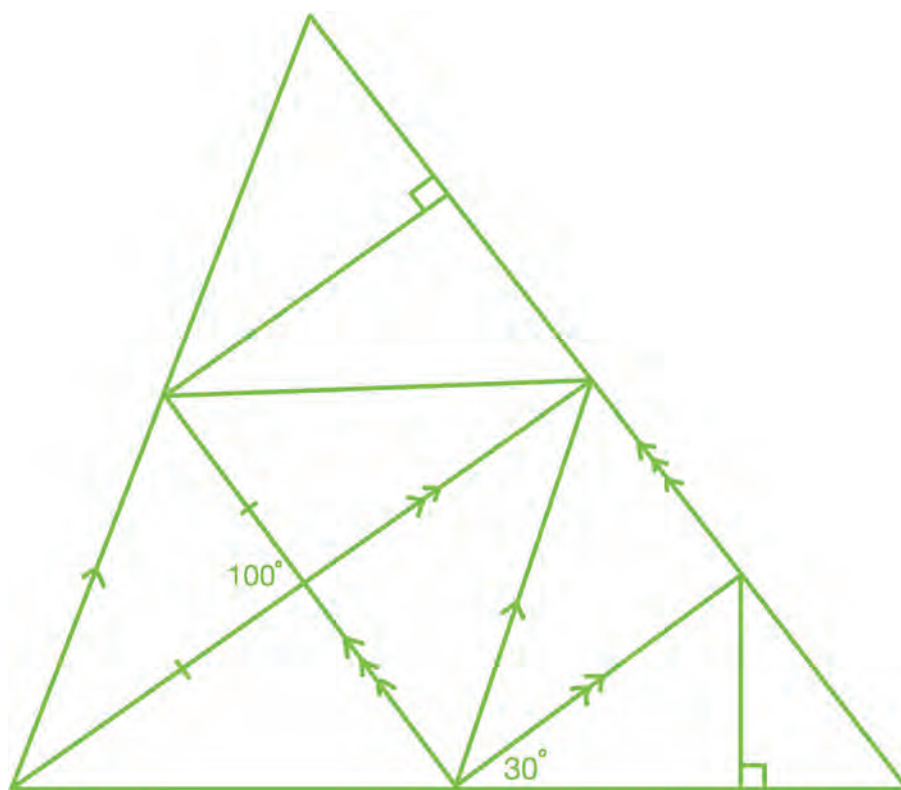
$$9x - 83 = 45 + 1x \quad \text{creates } 8x = 128$$

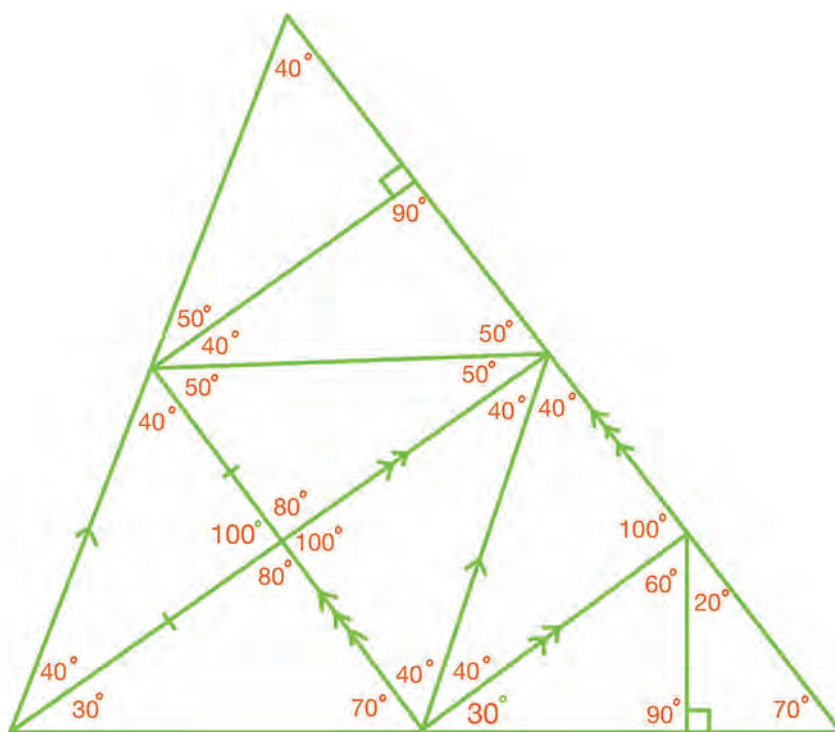
Activity D: Puzzle—Angles in a Triangle

Find the angle measure for every angle in this picture. Assume that lines marked parallel are parallel and lines marked congruent are congruent.



Hint: You will learn later in this chapter that in an isosceles triangle the base angles across from the congruent legs are congruent to each other.



Solution:**SHARE YOUR WORK**

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 4.1–4.3 (showing handwritten computations and corrections)
- Choice of activity (labeled with the title of the activity):
 - Activity A: Integrated Review
 - Activity B: Which One Doesn't Fit? Corresponding Parts of Triangles
 - Activity C: Fill in the Blank—Triangle Sum and Exterior Angles
 - Activity D: Puzzle—Angles in a Triangle

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson

4

Part 2: Triangles and Congruence

Lesson Introduction

Suggested time: 1.5 weeks

Lesson 4 continues with part 2. Refer to part 1 for learning objectives.

Exercise Sets

Read and/or watch instructional videos for the following sections, and complete the accompanying problem sets. Plan to complete a portion each day. As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 4.4, “Congruent Triangles by ASA and AAS” (179), and then complete the following problems in Exercise Set 4.4.
 - 1–39 odd
2. Read section 4.5, “Proofs Using Congruent Triangles” (186), and then complete the following problems in Exercise Set 4.5.
 - 1–35 odd
3. Read section 4.6, “Isosceles, Equilateral, and Right Triangles” (195), and then complete the following problems in Exercise Set 4.6.
 - 1–47 odd
 - Extension 49–55 odd

ASSIGNMENT CHECKLIST

- Read sections 4.4–4.6 and complete the assigned problems.
- Complete the chapter 4 test.
- Choose an activity to complete:

Activity A: Explore with Technology Triangle Congruence

Activity B: Investigate Proof Writing

Activity C: Challenge Yourself! Overlapping Triangles in a Detour Proof

Activity D: Error Analysis of Assuming Information from Diagrams

4. Optional: If you would like more practice, you have the option of completing the following, doing as many problems as needed.
- ☐ Chapter 4 Review and Vocabulary Check (199)
 - ☐ Chapter 4 Standardized Test Practice (202)

Chapter Test

In your textbook, complete the chapter 4 test on page 201. After completing the test, you or a supervising adult will grade it and mark the score at the top (for instance, 18/20). Then, review any mistakes and make necessary corrections.

Activities

Choose one of the following activities to complete.

- Activity A: Explore with Technology Triangle Congruence
- Activity B: Investigate Proof Writing
- Activity C: Challenge Yourself! Overlapping Triangles in a Detour Proof
- Activity D: Error Analysis of Assuming Information from Diagrams

Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	

	Notes
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

Activity A: Explore with Technology Triangle Congruence

In this chapter, you have learned to prove two triangles are congruent using the SSS, SAS, ASA, AAS, and HL theorems. Perhaps you have noticed that there are a couple of combinations that are not on the list! Have you wondered why?

Explore these two GeoGebra activities to investigate SSA and AAA and triangle congruence. (All online resources can be accessed at oakmeadow.com/curriculum-links.)

“SSA Exploration”

Note: There are links in the activity description to applets for SAS, ASA, and SSS. Feel free to explore why those theorems work and play around with visual models!

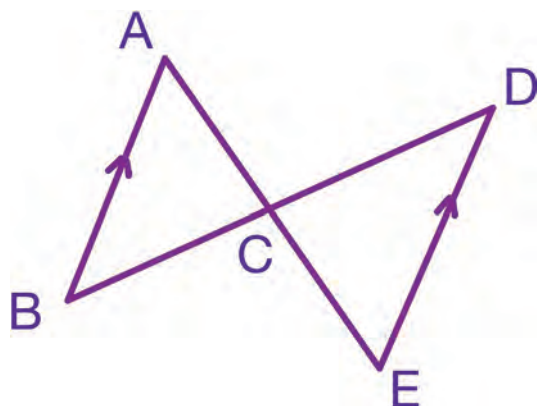
“AAA Exploration”

1. What did you observe about SSA?

Students should observe that having two congruent sides and an angle not between them cannot guarantee that two triangles are congruent. The angle that is in between the two congruent sides does not need to be congruent and will, therefore, open up many possibilities for the length of the third side. Therefore, two triangles with SSA congruent parts are not necessarily congruent.

2. What did you observe about AAA?

Students should observe that having all three angles congruent does not guarantee that two triangles are congruent. Angle measure has no bearing on side length, therefore two triangles with three congruent angles can have three completely different side lengths and are, therefore, not necessarily congruent.

Activity B: Investigate Proof Writing

Oftentimes, there is more than one way to correctly complete a proof. Ideally, we want to take the shortest path possible to reach the objective, but even then, several approaches may get you to your goal.

Solve the following proof two different ways.

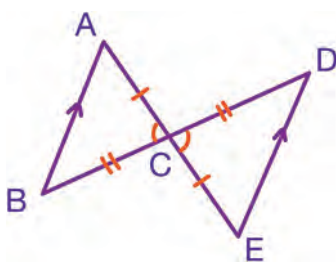
Given: $AB \parallel DE$

C is the midpoint of AE and BD

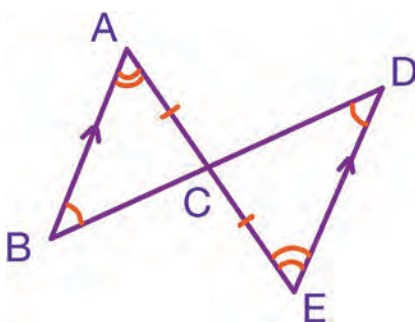
Prove: $AB \cong DE$

There are at least five ways to prove this. Students should demonstrate two distinct approaches. Verify the student's proofs.

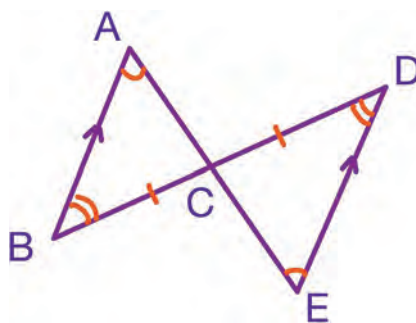
Proof 1:



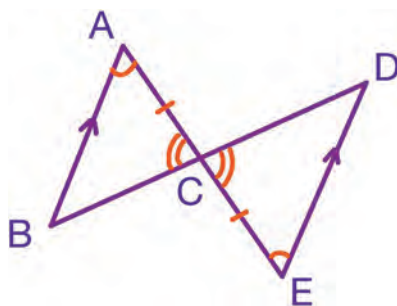
STATEMENTS	REASONS
1. $AB \parallel DE$	1. Given
2. C is the midpoint of AE and BD	2. Given
3. $AC \cong CE, BC \cong CD$	3. Definition of midpoint
4. $\angle ACB \cong \angle ECD$	4. Vertical angles \cong
5. $\triangle ACB \cong \triangle ECD$	5. SAS (3, 4, 3)
6. $AB \cong DE$	6. CPOCTAC

Proof 2:

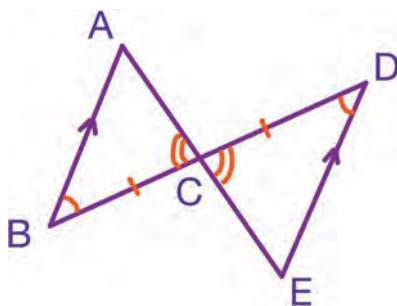
STATEMENTS	REASONS
1. $AB \parallel DE$	1. Given
2. C is the midpoint of AE and BD	2. Given
3. $\angle A \cong \angle E$, $\angle B \cong \angle D$	3. If \parallel lines then alt. Int. angles \cong
4. $AC \cong CE$	3. Definition of midpoint
5. $\triangle ACB \cong \triangle ECD$	5. AAS (3, 3, 4)
6. $AB \cong DE$	6. CPOCTAC

Proof 3:

STATEMENTS	REASONS
1. $AB \parallel DE$	1. Given
2. C is the midpoint of AE and BD	2. Given
3. $\angle A \cong \angle E$, $\angle B \cong \angle D$	3. If \parallel lines then alt. Int. angles \cong
4. $BC \cong CD$	3. Definition of midpoint
5. $\triangle ACB \cong \triangle ECD$	5. AAS (3, 3, 4)
6. $AB \cong DE$	6. CPOCTAC

Proof 4:

STATEMENTS	REASONS
1. $AB \parallel DE$	1. Given
2. C is the midpoint of AE and BD	2. Given
3. $\angle A \cong \angle E$	3. If \parallel lines then alt. Int. angles \cong
4. $AC \cong CE$	3. Definition of midpoint
5. $\angle ACB \cong \angle ECD$	4. Vertical angles \cong
5. $\triangle ACB \cong \triangle ECD$	5. ASA (3, 4, 3)
6. $AB \cong DE$	6. CPOCTAC

Proof 5:

STATEMENTS	REASONS
1. $AB \parallel DE$	1. Given
2. C is the midpoint of AE and BD	2. Given
3. $\angle B \cong \angle D$	3. If \parallel lines then alt. Int. angles \cong
4. $BC \cong CD$	3. Definition of midpoint
5. $\angle ACB \cong \angle ECD$	4. Vertical angles
5. $\triangle ACB \cong \triangle ECD$	5. ASA (3, 4, 3)
6. $AB \cong DE$	6. CPOCTAC

Activity C: Challenge Yourself! Overlapping Triangles in a Detour Proof

Complete the following proof.

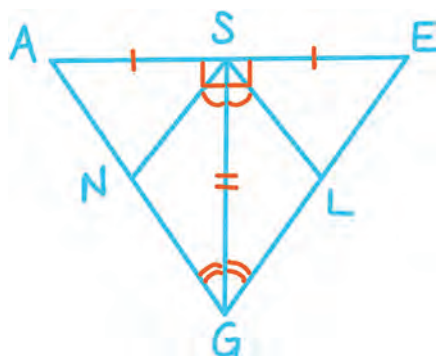
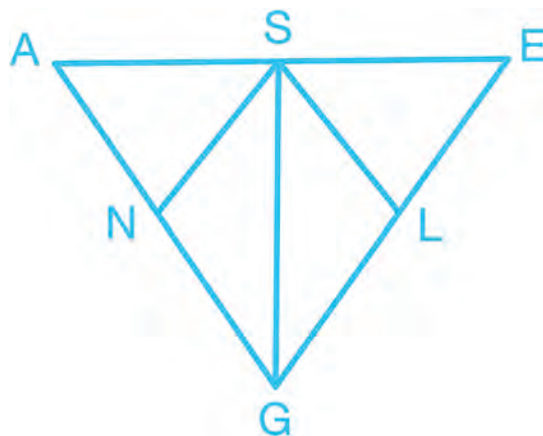
Given: SG bisects $\angle NSL$

$SG \perp AE$

S is the midpoint of AE

Prove: $\angle SNG \cong \angle SLG$

Verify the student's proof. One possible solution is given:



STATEMENTS	REASONS
1. $SG \perp AE$	1. Given
2. $\angle ASG = 90^\circ, \angle ESG = 90^\circ$	2. Definition of \perp
3. $\angle ASG \cong \angle ESG$	3. Substitution
4. S is the midpoint of AE	4. Given
5. $AS \cong SE$	5. Definition of midpoint
6. $SG \cong SG$	6. Reflexive Property
7. $\triangle ASG \cong \triangle ESG$	7. SAS (5, 3, 6)
8. $\angle SGN \cong \angle SGL$	8. CPOCTAC
9. SG bisects $\angle NSL$	9. Given
10. $\angle NSG \cong \angle LSG$	10. Definition of bisects
11. $\triangle GNS \cong \triangle GSL$	11. ASA (8, 6, 10)
12. $\angle SNG \cong \angle SLG$	12. CPOCTAC

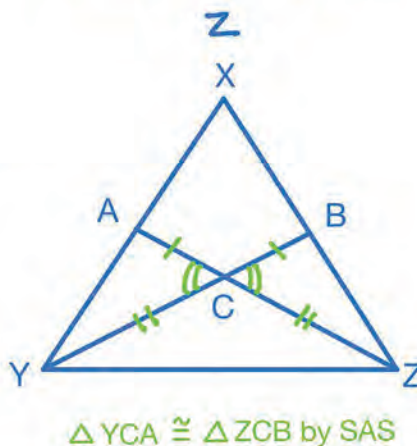
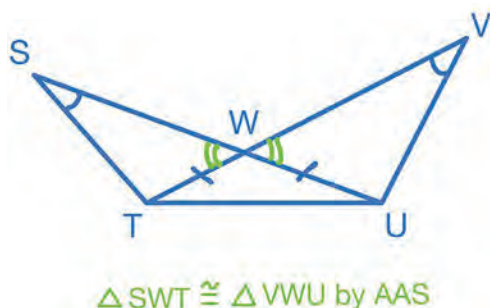
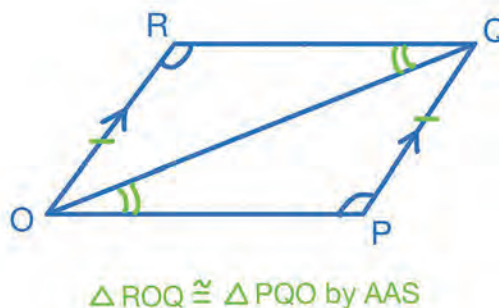
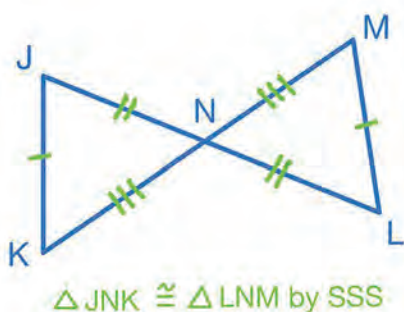
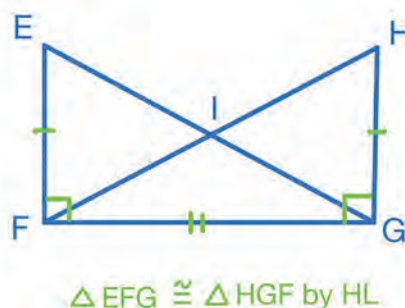
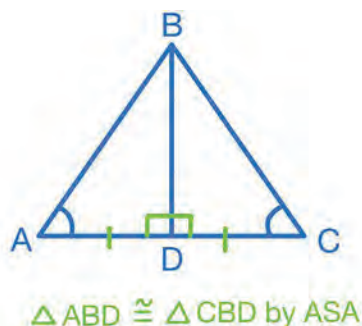
Activity D: Error Analysis of Assuming Information from Diagrams

It can sometimes be tempting to assume a lot of information from a given diagram, but it is important to remember that there are only a few things we can ever assume to be true without additional information.

A group of students were given the following diagrams and asked to mark the diagram and write any triangle congruent statements. **Only one diagram is correct.**

Note: Anything blue was given in the diagram. All green markings were added by the students.

1. Explain what incorrect assumptions were made in each diagram.



Students should explain the errors in each diagram.

First row, first diagram: The student incorrectly assumed that D was a midpoint and that BD was perpendicular to AC . We cannot assume midpoints and perpendicular lines. They should have used the reflexive property to mark the shared line and the converse of the isosceles triangle theorem to mark AB and CB congruent.

First row, second diagram: The student incorrectly assumed right angles and that EF was congruent to HG . We cannot assume right angles or that segments that look congruent are congruent. The student did correctly use the reflexive property to mark FG congruent to itself. The only other thing that can be assumed in this diagram is the vertical angles.

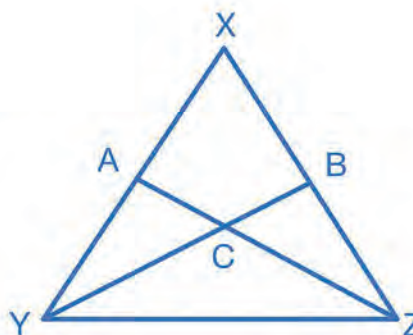
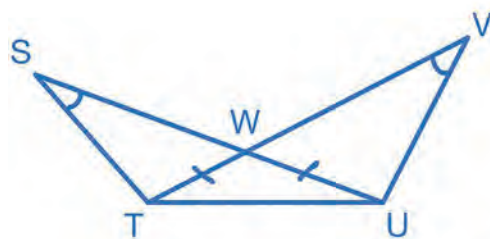
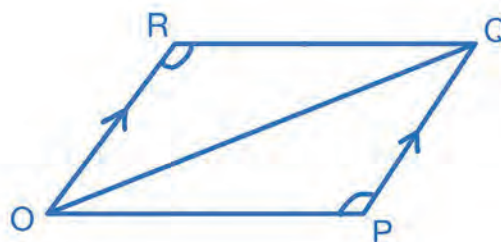
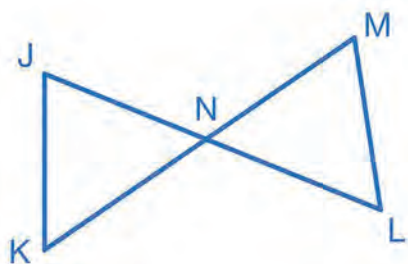
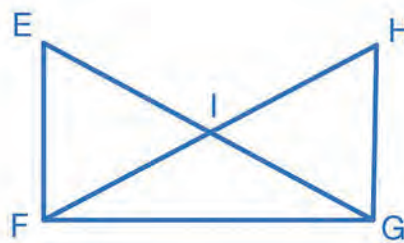
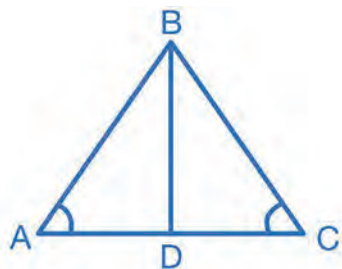
Second row, first diagram: The student incorrectly assumed that corresponding sides were congruent. We cannot assume that sides that appear to be congruent are. The only thing that can be assumed in this diagram is the vertical angles.

Second row, second diagram: The student incorrectly assumed that parallel segments are congruent. They also marked the wrong alternate interior angles congruent. The marked angles would be congruent on the transversal if RQ was parallel to OP . The student should have marked the correct pair of alternate interior angles congruent and used the reflexive property to show OQ congruent to itself.

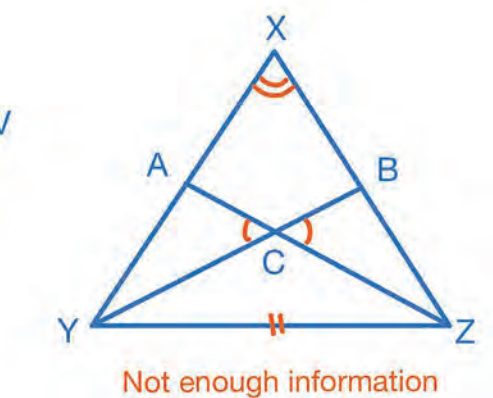
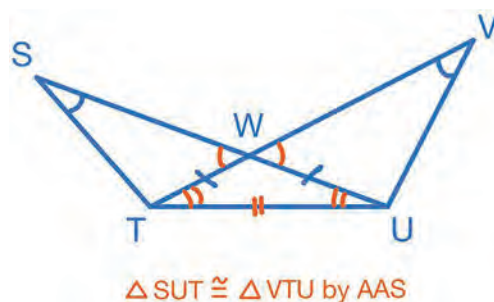
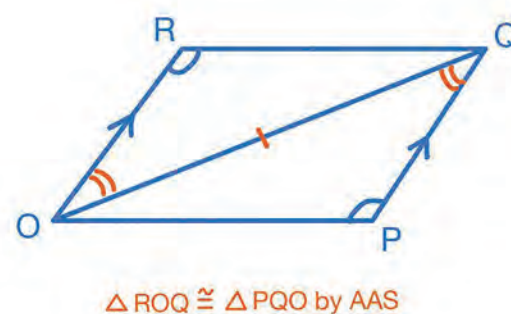
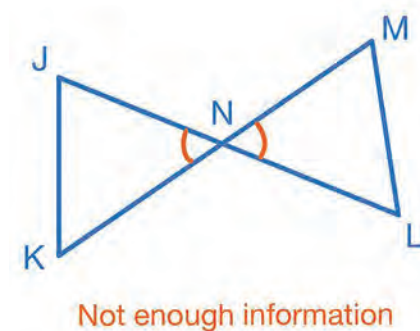
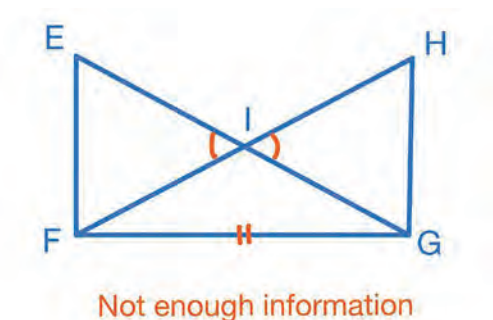
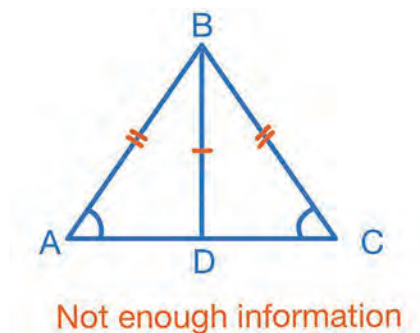
Third row, first diagram: The student is correct! They correctly marked vertical angles and concluded the triangles were congruent by AAS. The only other thing they could have assumed from the diagram was that TU is congruent to itself through the reflexive property.

Third row, second diagram: The student incorrectly assumed that corresponding line segments were congruent. We cannot assume that segments that appear to be congruent are. The student did correctly mark the vertical angles. The other things that can be correctly assumed from this diagram are the other pair of vertical angles, that angle X is congruent to itself through the reflexive property, and that YZ is congruent to itself by the reflexive property.

2. Mark each diagram correctly and make any congruence statements, if possible. If not enough information exists to draw a conclusion, say “not enough information.”



Correct work is below. Students can make an additional correct conclusion about the first diagram in the third row.



- Write a generalized statement or rule that you could give to help someone avoid incorrect assumptions.

Students should state that the only things that can be assumed in a diagram are vertical angles and shared sides and angles through the reflexive property. All other assumptions require additional information.

SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 4.4–4.6 (showing handwritten computations and corrections)
- Chapter 4 test
- Choice of activity (labeled with the title of the activity):
 - Activity A: Explore with Technology Triangle Congruence
 - Activity B: Investigate Proof Writing
 - Activity C: Challenge Yourself! Overlapping Triangles in a Detour Proof
 - Activity D: Error Analysis of Assuming Information from Diagrams

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.